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THE ORBIT OF ARIEL 2 (1964-15A)- THE FIRST TWELVE MONTHS

by

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FARNBOROUGH HANTS

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SUMMARY

The definitive orbit for Ariel 2 (1964-15A) is computed, from Minitrack observations, for a period of twelve months from the launch of the satellite. The orbit is described by a model with eight orbital parameters and these parameters are listed at every twenty-fifth nodal passage. The angular observations are accurate to about 1' and, as a result, the average computed standard deviations of the eight fitted orbital parameters are as follows: 1 m in semi-major axis, 10^{-5} in eccentricity, 2" in inclination, 4" in right ascension of the node, 30" in argument of perigee, $0^s.03$ in time at the node, and 0.001 deg/d^2 and 0.001 deg/d^3 in the linear and quadratic coefficients occurring in the mean motion polynomial.

Ephemerides computed from the listed orbital parameters will be accurate to about $\frac{1}{2}$ km, the accuracy required by the Ariel 2 experimenters. Limitations which prevent the accuracy from being better than this are discussed.

Departmental Reference: Space 122

CONTENTS

1 INTRODUCTION

Ariel 2 is the second of the series of satellites being launched in the scientific programme based on Anglo-American co-operation. As with Ariel 1, the first of the series, the satellite was constructed and launched by the U.S. (NASA). Whereas British responsibility for Ariel 1 was confined to the scientific experiments (including telemetry data analysis), for Ariel 2 it has been extended to the determination of definitive orbital parameters. Responsibility of the U.K. will continue to grow for the third satellite of the series, at present styled U.K.3 and due to be launched early in 1967, since this will be the first spacecraft actually to be built in Britain.

Three experiments constituted the scientific payload of Ariel 2, each sponsored by a scientific establishment in the U.K.: measurement of galactic noise by the Mullard Radio Astronomy Laboratory, Cambridge; measurement of micrometeorite flux and particle sizes by the Nuffield Radio Astronomy Laboratory, Jodrell Bank; and measurement of atmospheric ozone by the Meteorological Office, Bracknell. The satellite, known before launch as S.52 or U.K.2 and after launch as Ariel 2 or 1964-15A, was successfully placed in orbit at 17.25 UT on 27th March 1964 from the NASA Wallops Station, Virginia, by a four-stage solid-propellant rocket. The experiments continued to work until the end of September 1964, by which time the spin rate of the satellite was too low for useful data to be obtainable.

Tracking data have been provided by the Minitrack network of NASA (STADAN)[1]. The first observations were from Lima, Peru, within 4 hours of the time of launch, and data are still (November, 1965) being obtained from the network, though it is expected that NASA will officially terminate the project in the near future. The data have been analysed in Space Department, R.A.E., and the main purpose of this Report is to tabulate the orbital parameters derived. These parameters have been calculated by use of the standard programmes[2,3] for the Pegasus computer and are given at intervals of 25 ascending nodes (about $1\frac{3}{4}$ days). Computation of orbital parameters was stopped after the orbit had been analysed for twelve months.

2 MINITRACK OBSERVATIONS

The STADAN network consists of a dozen Minitrack stations distributed as shown in Fig.1. The inclination of the Ariel 2 orbit being too low for data from Alaska, observations were obtained from the eleven stations listed in Table 1. The Table gives latitude, longitude and height relative to the Fischer ellipsoid[4], the currently best world-wide geodetic datum. Also given are the number of observations used from each station.

A total of about 3700 observations has been used over the twelve month period, corresponding to about ten per day. Of these, about 100 were rejected during analysis due to the size of residuals, but some of the rejections were due to incorrect punching of tapes before analysis so the data are in fact very reliable.

An accuracy of the order of a millisecond is claimed for the quoted times of observations, and time errors have been ignored in the analysis. Experience with Ariel 1 showed[5] that the angular accuracy is effectively about 1 minute of arc and this value was used in the analysis. It covers errors in the basic interferometer measurements, ionospheric refraction correction, station co-ordinates and orbital model, and can also be regarded as incorporating the explicitly ignored time errors.

For the first 36 hours after launch, observations with elevations as low as 20° were made and have been used to obtain the first set of orbital parameters. To minimise unknown refraction errors, data made available after this initial short period have been confined to observations with high elevations; more than half the observations have had elevations greater than 80° and there have been only four with elevations less than 60°.

The coverage of each interferometer essentially consists of a pair of narrow fans in the vertical plane, one north-south and the other east-west, only one being used on a given pass. For this reason the azimuths of most observations have been close to 0°, 90°, 180° or 270°. A station supplies, for a given pass, one or two observations. If there are two, since they are both in the same narrow fan, they are only about 10 to 15 seconds apart in time.

3 ANALYSIS OF THE OBSERVATIONS

3.1 Dynamic model of the orbit

The analysis has been carried out using the standard R.A.E. orbit improvement technique[2,3] based on Merson's smoothed elements[6]. The orbital model chosen for Ariel 2 has twelve parameters associated with each of the epochs defined by every 25th ascending node. Eight of the twelve parameters are determined by differential correction of approximate values; these eight are: a_o (semi-major axis), e_o (eccentricity), i_o (inclination), Ω_o (right ascension of the defining node), ω_o (argument of perigee), t_o (time at the defining node) and the parameters n_1 and n_2 such that the mean motion at time t is given by:-

$$n = (\mu/a_o^3)^{\frac{1}{2}} + n_1 (t - t_o) + n_2 (t - t_o)^2 ,$$

where $\mu = 398\ 602\ km^3/sec^2$. The remaining four parameters - denoted by e_1, i_1, Ω_1 and ω_1 - give time-linear contributions to e, i, Ω and ω; unlike n_1 and n_2, they are not fitted to the observations but are given pre-calculated values. They represent drag, luni-solar and earth-gravity (J_3, J_4 and J_2^2 terms) perturbations of the orbit, and are valid for only two or three days each side of the epoch t_o. It must be emphasised that Ω_1 and ω_1 do not include the large secular terms which arise from the J_2 perturbation; these are automatically allowed for by the model. Similarly, e_1 does not represent the main variation of e due to drag, this being allowed for through the parameter n_1. Some printed sets of parameters have been circulated in which coefficients Ω_2 and ω_2 were included in addition to Ω_1 and ω_1. However their values were so small as to be quite without significance (see also Section 5).

Ref.3 gives the complete set of formulae by means of which geocentric position is computed from the twelve orbital parameters. They differ from the formulae of the original Pegasus programme[2] in two respects. First, the position of the satellite at time t is computed as if the time was actually $t + \delta t$ where

$$\delta t = 0.088 \sin 2 (\theta - \Omega - 18^o) \text{ sec} \quad ,$$

θ being the sidereal time. This formula gives a first-order approximation to the 12-hour along-track oscillation of the satellite caused by the tesseral harmonics - in particular by $J_{2,2}$ - of the earth's gravitational field. Further details are given in the Appendix. Naturally, the topocentric position of the satellite is found by taking the station co-ordinates (in space axes) at the true time t.

The other divergence from the formulae of the original Pegasus programme is that a term has been introduced into the expression for mean anomaly, M, associated with the parameters Ω_1 and ω_1. The full formula for M is

$$M = M_o + (n_o - \omega_1 - \Omega_1 \cos i_o)(t - t_o) + \tfrac{1}{2} n_1 (t - t_o)^2 + \tfrac{1}{3} n_2 (t - t_o)^3 \ ,$$

where M_o is determined from e_o and ω_o; t is taken, of course, to include the 12-hour δt term already mentioned. n_o denotes $(\mu/a_o^3)^{\frac{1}{2}}$ and the formula for n given earlier is not affected.

3.2 Computer operation

From the receipt of observations from NASA to the generation of a given set of definitive orbital parameters, there are seven programmes which have to

be run on the Pegasus computer. Allowing for the fact that the differential correction programme is used twice, the seven programmes correspond to eight operating stages as listed below. The first two stages apply to the observations in general and the remaining six to the analysis at a particular node.

(i) The direction cosines, of which the data consist, are punched on paper tape; they are converted to azimuth and elevation by the computer, which provides output in a form suitable for re-input with the next programme.

(ii) Observations, sometimes several hundred at a time, are stored on the Ariel 2 M.T.L. (magnetic tape library).

(iii) The initial parameters at a given node are predicted by extrapolation from the definitive parameters 25 nodes before (except for the first definitive node when NASA estimates are used).

(iv) About 30 observations, chosen by the operator for the analysis at the given node, are transferred from the M.T.L. to an Ariel 2 S.D.T. (selected data tape); some preliminary processing is incorporated in the selection programme.

(v) A single iteration of the main differential correction programme is run; the parameters - and in particular n_1 - are then sufficiently accurate for the next stage.

(vi) Contributions to e_1, i_1, Ω_1 and ω_1 due to gravity (J_3, J_4 and J_2^2 terms) and drag perturbations are found; these are functions of the orbital parameters and the drag terms depend on n_1.

(vii) The luni-solar contributions to e_1, i_1, Ω_1 and ω_1 are found and added in.

(viii) As many further iterations as necessary (normally two) of the differential correction programme are run, e_1 etc being held fixed; after the last iteration the standard deviations of a_o, e_o, i_o, Ω_o, ω_o, t_o, n_1 and n_2 are obtained.

Stage (ii) requires virtually the same computer time - about 40 min - irrespective of the number of observations involved (within the capacity of the M.T.L.). For the other stages the average times at the computer, based on a set of 30 observations, are as follows: for stage (i) 5 min; for (iii) 2 min; for (iv) 8 min; for (v) 15 min; for (vi) 1 min; for (vii) 1 min; for (viii) 30 min. Thus the total computing time for the analysis at each node worked out at about 70 min under ideal conditions. An actual average was

nearer 90 min allowing for delays due to the rejection of observations etc. The time away from the machine, including a little more than 1 hr for punching 30 observations, was about another 90 min. Since analysis was conducted at 210 different nodes the routine work required grand totals of about 320 Pegasus hours and 650 man hours.

3.3 Miscellaneous points

Times of observations were provided relative to the WWV time transmissions from America. Since the experimenters were using telemetry data relative to WWV it was decided not to correct to any of the standard time systems (universal, ephemeris or atomic time). If it is so desired, the orbital parameters in this paper may be corrected to datum UT2 as follows, only Ω_0 and t_0 being affected: to t_0 add the time (in seconds) by which WWV is slow on UT2; to Ω_0 add $0^\circ.004 \times$ (UT2-WWV) where the time difference is in seconds.

The analysis at each defining node has used about 30 observations, spanning a period of 3 to 4 days centred on the node. Since the defining nodes are only $1\frac{3}{4}$ days apart there is considerable overlap, about half the observations being used twice. This was done deliberately since in any application of the 'fitting by moving arcs' method a narrow overlap is a source of inaccuracy. Accuracies actually obtained during the periods of overlap are considered in Section 5.

A difficulty arose in the analysis of the same observations at two different nodes since WWV time was set back $0^s.1$ at 1965 January 1.0. Each observation close to this time of discontinuity had to be duplicated in the library of Ariel 2 observations, once with the original observation time and again with an adjusted time, the observation selected depending on whether the defining node used was, respectively, the same side of the discontinuity as the observation or the opposite side.

4 RESULTS

Orbital parameters for Ariel 2 are listed in Table 2. Successive columns of the Table are as listed below, zero suffixes being omitted for convenience.

Node number, m

Date

Time

Semi-major axis, a (km)

Eccentricity, e

10000 e_1, with e_1 in units/100 days

a (1 - e)

Inclination, i (degrees)

100 i_1, with i_1 in degrees/100 days

Right ascension of the node, Ω (degrees)

100 Ω_1, with Ω_1 in degrees/100 days

Argument of perigee, ω (degrees)

ω_1 (degrees/100 days)

Mean anomaly, M (degrees)

Mean motion, n (degrees/100 days)

n_1 (degrees/(100 days)^2)

10^{-2} n_2, with n_2 in degrees/(100 days)^3

Number of observations used, N

Extent of the observations, D (days)

Standard deviation of an observation of unit weight, ε

Modified Julian Day representation of date/time, MJD.

The nodes are numbered such that m would have been zero at a fictitious node half an hour before launch. The quantities $a(1 - e)$, M and n are useful derived parameters. Although $a(1 - e)$ gives an indication of the perigee distance, r_p, it is not exactly equal to r_p. To $O(J_2)$ the perigee distance is given by:-

$$r_p = a (1 - e) + \frac{1}{4} J_2 \frac{R^2}{p} \{\sin^2 i \cos 2 \omega - (1 - e)(2 - 3 \sin^2 i)\} .$$

The values of the eight differentially-corrected parameters are followed, in Table 2, by their computed standard deviations, the unit in each case being that of the final figure quoted for the parameter. The quantity ε is a measure of the goodness of fit. Its expected value is 1 (non-dimensional), corresponding to the adopted weighting of **observations** with standard deviation 1'; if ε is, say, 2.5 the fit is such that the observations must, a posteriori, be regarded as only accurate to $2\frac{1}{2}$'. Every standard deviation includes ε as a factor. If this factor were removed, the remaining quantity would be independent of fit and would depend on, and therefore provide an indication of, the geometrical coverage of the orbit by the relevant station observations.

It is seen that average values for the standard deviations of the eight basic parameters are as follows:-

σ_a	1 metre	σ_ω	$0^\circ.007$
σ_e	0.00001	σ_{t_o}	$0^s.03$
σ_i	$0^\circ.0005$	σ_{n_1}	$12^\circ/(100 \text{ days})^2$
σ_Ω	$0^\circ.001$	σ_{n_2}	$1500^\circ/(100 \text{ days})^3$.

The remarkable accuracy assessed for semi-major axis arises from the relation $n^2 a^3 = \mu$ (Kepler's third law). It is really n that is being measured so accurately and the values of a may be systematically in error by as much as 20 m due to error in the adopted value for μ, viz. 398 602 km^3/sec^2. The above σ_a corresponds, in fact, to $\sigma_n = 0^\circ.1/100$ days or a maximum contribution to σ_M (when $1\frac{3}{4}$ days away from t_o) of about $0^\circ.002$. A better assessment of orbital distance accuracy is given by $\sigma\ (a\ \overline{1 - e}) = 70$ metres. It can readily be checked that the values given for σ_{n_1} and σ_{n_2} also both correspond, for $t - t_o = 1\frac{3}{4}$ days, to contributions of about $0^\circ.002$ to σ_M. Thus an approximate estimate of the maximum along-track error in position computed from the definitive orbital parameters may be obtained from $\sqrt{3} \times 0^\circ.002$ at 7000 km; this estimate is less than $\frac{1}{2}$ km. Further discussion of errors in computed position is to be found in Section 5.

To indicate the behaviour of the basic parameters relative to their tabulated standard deviations, Figs.2-10 have been plotted. The idea behind these plots has been, as far as possible, to use a scale large enough for the representation of each fitted value of a parameter by a vertical line of length 2σ centred on the fitted value. To this end secular terms have been removed from some of the parameters by preliminary fitting of some suitable polynomial, and for two of the parameters - e and ω - the long-periodic variation has also largely been removed.

For i, n_1 and n_2 a suitable scale is available without modification of the data from Table 2; Figs.5, 9 and 10 represent these three parameters, the plotted lines being of length $2\sigma_i$, $2\sigma_{n_1}$ and $2\sigma_{n_2}$ respectively. Eccentricity, right ascension of the node and argument of perigee have been represented in Figs.4, 6 and 7 by graphs of Δe, $\Delta\Omega$ and $\Delta\omega$ respectively, where

$$\Delta e = e + 1.08 \times 10^{-6}\, m - 7 \times 10^{-4} \sin \omega \;,$$

$$\Delta\Omega = \Omega + k \times 360^{\circ} + 0^{\circ}.287515\, m + 2^{\circ}.29 \times 10^{-7}\, m^{2} + 1^{\circ}.18 \times 10^{-11}\, m^{3}$$

and

$$\Delta\omega = \omega + k \times 360^{\circ} - 0^{\circ}.21385\, m - 2^{\circ}.3 \times 10^{-7}\, m^{2} - 0^{\circ}.4 \cos \omega \;;$$

here k is an integer which has been introduced to avoid the discontinuities of 360° in the tabulated values of Ω and ω.

Figs.2 and 3 both relate to semi-major axis. The values of σ_a are so small that the scale can not be made large enough over the whole range of a, even after removal of a very-high-order polynomial. Fig.2 corresponds to the removal of a quintic, and plots $\Delta_1 a$, where

$$\Delta_1 a = a + 1.61 \times 10^{-2}\, m - 1.288 \times 10^{-5}\, m^{2} + 6.915 \times 10^{-9}\, m^{3} - 1.501 \times 10^{-12}\, m^{4} + 1.159 \times 10^{-16}\, m^{5} .$$

In order that a graph showing standard deviations might be drawn, the data were divided into seven sections and a separate polynomial (quartic) removed from each section, the constant terms being adjusted so that the polynomials joined up. The resulting graph is given in Fig.3 by the plot of $\Delta_2 a$, where, in km,

$$\Delta_2 a = a + 2.517 \times 10^{-2}\, m - 3.854 \times 10^{-5}\, m^{2} + 3.301 \times 10^{-8}\, m^{3} - 1.082 \times 10^{-11}\, m^{4} \qquad \text{for } 0 < m < 750 \;,$$

$$\Delta_2 a = a + 8.4985 - 9.22 \times 10^{-3}\, m + 1.539 \times 10^{-5}\, m^{2} - 6.976 \times 10^{-9}\, m^{3} + 1.277 \times 10^{-12}\, m^{4} \qquad \text{for } 750 < m < 1500 \;,$$

$$\Delta_2 a = a - 180.9639 + 4.424 \times 10^{-1}\, m - 3.84 \times 10^{-4}\, m^{2} + 1.48277 \times 10^{-7}\, m^{3} - 2.1107 \times 10^{-11}\, m^{4} \qquad \text{for } 1500 < m < 2250 \;,$$

$$\Delta_2 a = a - 1142.1030 + 1.83778\, m - 1.092788 \times 10^{-3}\, m^{2} + 2.88134 \times 10^{-7}\, m^{3} - 2.82585 \times 10^{-11}\, m^{4} \qquad \text{for } 2250 < m < 3000 \;,$$

$$\Delta_2 a = a + 43.1415 - 4.1146 \times 10^{-2} m + 2.0548 \times 10^{-5} m^2$$
$$- 3.4637 \times 10^{-9} m^3 + 1.9394 \times 10^{-13} m^4 \qquad \text{for } 3000 < m < 3750 ,$$

$$\Delta_2 a = a + 375.2463 - 4.7361 \times 10^{-1} m + 2.24885 \times 10^{-4} m^2$$
$$- 4.53198 \times 10^{-8} m^3 + 3.34635 \times 10^{-12} m^4 \qquad \text{for } 3750 < m < 4500 ,$$

and

$$\Delta_2 a = a - 22497.1421 + 18.545985\, m - 5.723216 \times 10^{-3} m^2$$
$$+ 7.845224 \times 10^{-7} m^3 - 4.027167 \times 10^{-11} m^4 \qquad \text{for } 4500 < m < 5250 .$$

Fig.8 relates to the remaining parameter, t_o. This is plotted in the Modified Julian Day number form, a quartic polynomial being removed from the values of Table 2. Again standard deviations cannot be represented and this time they are so small relative to the scale of the graph - the average σ_{t_o} being 3.5×10^{-7} day - that division of the data into sections would not help. The quantity plotted in Fig.8 is Δt_o, where, in days,

$$\Delta t_o = t_o - 7.0318 \times 10^{-2} m + 4.1 \times 10^{-8} m^2 + 4.06 \times 10^{-12} m^3 - 1.27 \times 10^{-16} m^4 .$$

Although the polynomial (and sinusoidal) expressions removed from the orbital parameters have been listed above, it is not claimed that the polynomials represent the real behaviour of the parameters or that the shapes of the residual plots are significant. As already stated, the polynomials have only been removed in order that the magnitudes of the standard deviations should be visible, on the graphs, against the background of the fluctuation of the parameters. The graphs indicate certain facts about interpolation that are discussed in the next paragraph.

Suppose that the orbital parameters corresponding to a certain node had not been available and that they were estimated by a suitable form of interpolation in the adjacent sets of parameters. Then Figs.4, 5, 6 and 7 show that the values interpolated for e, i, Ω and ω, respectively, would be very good; it would be almost certain, in fact, that any interpolated value would be within two average standard deviations of the true parameter. For a and n_1, however, it is clear from Figs.3 and 9 that such accurate interpolation would be impossible and for t_o the situation would be worse still. (The simplest thing to do with the final

parameter, n_2, would be to set it zero - Fig.10 shows that the error would be no greater than in interpolating n_1.)

Inadequacy of interpolation for the parameters a, t_0 and n_1, due to irregular variation in air drag, explains why it was necessary to obtain orbital parameters as frequently as at 25 node intervals (see also Section 5) and suggests that an even finer analysis, say at 10 or 15 node intervals, might have given further information on drag fluctuations.

5 ACCURACY OF POSITION COMPUTATION

From a set of orbital parameters derived on the basis of a given orbital model, the position of a satellite at any time may be computed using the same orbital model; indeed, for satellites transmitting scientific measurements, the provision of an ephemeris - or world map - is one of the chief motives for determining orbital parameters. In the case of Ariel 2 the experimenters wanted positions to be normally accurate to $\frac{1}{2}$ km or better, and almost never worse than 1 km.

Successive orbit determinations were initially carried out at 50-node intervals, starting from node 25. Allowing for overlap this meant that a given set of parameters had to be valid for a period stretching from about 26 nodes before the relevant defining node to about 26 nodes after. A useful measure of the accuracy of position computation was available by comparing the positions at various times - in particular during the overlap period - as computed from two sets of parameters, those at the previous defining node and those at the following one.

Let d be the distance between two estimates of satellite position as above, d being a function of time. The behaviour of d might be expected to be compounded of a periodic term and a secular term. This is illustrated in Fig.11, based on orbital parameters for defining nodes 75 and 125. The lower plot shows the fine behaviour of d over a single revolution - from node 100 to node 101. The upper plot shows the coarse behaviour between nodes 92 and 110; a curve has not been drawn because of the oscillation which occurs between the plotted points. It is clear at once that the desired accuracy is not being achieved.

It was largely as a result of contemplating Fig.11 that the decision was made to produce orbital parameters for the intermediate nodes 50, 100 etc. Fig.12 gives the coarse and fine behaviour of d for compared positions based on the parameters for nodes 75 and 100. Fig.13 gives a similar picture based on nodes 100 and 125. Although the true position of the satellite is, of course, never known, it is now very probable that the desired accuracy is being attained.

A second way of estimating the accuracy of position computation is to employ the covariance matrix of the orbital parameters used for this computation. A Pegasus programme is available for doing this and has been described elsewhere[7]. Let S be the root mean square of the distance between the true position of a satellite and its computed position from the parameters at a defining node (only one defining node is involved now). Fig.14 gives the coarse behaviour of S between nodes 1950 and 1968 based on the covariance matrix of the orbital parameters at node 1950. Fig.15 gives the fine behaviour of S between nodes 1962 and 1963; values based on covariance matrices from nodes 1950 and 1975 have been averaged here, but they scarcely differed. On the same graph is the corresponding plot of d, similar to that plotted in earlier figures.

The agreement between the S and d plots of Fig.15 is excellent, bearing in mind that they arise from quite different methods of accuracy assessment. Similar plots are given in Fig.16, starting from a different pair of nodes, 2275 and 2300. This time the agreement is much worse, an average value for d/S being about $3\frac{1}{2}$. If the errors in the position of the satellite, computed from two sets of parameters, were uncorrelated, the expected value of d/S would be $\sqrt{2}$, but with the parameters based on overlapping sets of data the expectation would be less than this. Thus a value $3\frac{1}{2}$ probably represents a systematic error in the orbital model. However, it is felt that even in this case the accuracy is unlikely to be worse than 1 km. It should be remarked that the choice of nodes upon which Fig.11 to 16 were based was a purely random one.

The effect of two possible sources of systematic error was investigated in a similar way to the above, using the quantity d. It has been remarked in Section 3.1 that parameters Ω_2 and ω_2 were originally included in the model but later dropped. The effect of dropping Ω_2 and ω_2 from the parameters for node 75, all other parameters remaining unchanged, is shown in Fig.17. This effect is, of course, proportional to $(t - t_o)^2$ and its maximum value may be seen to be, in metres, about 33 $(t - t_o)^2$, measuring time in days. This is completely negligible.

The Fischer ellipsoid was adopted early in the analysis, but without recomputing all previous sets of parameters obtained when the stations were not all referred to the same datum. One set of parameters was recomputed, the set for node 525. Fig.18 indicates the negligible change in position as computed from the parameters before and after the change.

6 COMPARISON WITH OTHER ORBITS OF ARIEL 2

Elements of Ariel 2 have been issued by two American computing centres, Spacetrack and NASA. In each case new sets of elements are produced, on average, about every nine days, some five times less frequently than the elements tabulated in this paper. For this reason, among others, the American orbits must be regarded as significantly less accurate than the definitive R.A.E. orbit. The Spacetrack bulletins, in any case, describe their data as for prediction purposes only and unsuitable for scientific analysis.

The NASA elements, like the R.A.E. elements, have been derived from Minitrack observations. However, in addition to being issued less frequently they differ from R.A.E. elements in that they are related to a different dynamic model of the orbit; also, each set corresponds to some arbitrary epoch and not to the time of a particular nodal passage. The precise definitions of NASA elements are rather difficult to discover and it is not certain that the same orbital model is used for the analysis of all satellites. It was therefore of interest to compare NASA elements - interpolated to the time of the nearest set of R.A.E. elements - with the R.A.E. elements, using the differences to establish the most likely interpretation of the NASA elements.

The comparison has been made and it seems that NASA must have used a set of elements which will be defined in the next paragraph. They are denoted by $\bar{a}$, $\bar{e}$ etc. to distinguish them from R.A.E. elements. Formulae are given in Sections 6.1 and 6.2 which relate the two sets of elements. On the assumption that the NASA elements are the barred elements, a number of sets have been transformed into Merson's elements for comparison with R.A.E. elements. The average difference is then about three or four times the R.A.E. standard deviations and - since the R.A.E. elements are the more accurate - the agreement is very satisfactory. This is illustrated by Fig.19 which gives the R.A.E. eccentricity graph; values of e from NASA are plotted before and after transformation and it is seen that after transformation they lie on or near the R.A.E. curve. Comparison of R.A.E. and NASA ephemerides is considered in Section 6.3 and illustrated by Fig.20.

The elements $\bar{a}$, $\bar{e}$ etc. are defined, to O(e), by a double averaging of osculating elements. The first averaging is with respect to mean anomaly and the second with respect to argument of perigee; these averagings remove, respectively, short-periodic perturbations associated with the J_2 term of the geopotential function and long-periodic perturbations associated with the J_3 term. Despite the general uncertainty over their orbital methods, there is reason to believe that NASA remove J_5 perturbations also, but these are much smaller than the J_3 perturbations and are not considered here.

The formulae given below have been derived by using the results of Merson[8], who compared his smoothed elements[6] with the mean elements of Kozai[9]. The elements $\bar{e}$, $\bar{i}$ and $\bar{\Omega}$ are very close to those which Kozai reached after removing long-periodic terms but the $\bar{a}$ and $\bar{\omega}$ defined here differ from those of Kozai since each of the latter was given a bias relative to the true average short-periodic perturbation.

The element a is dealt with after e, i, Ω and ω in the comparisons below and is then considered in conjunction with the mean motion n and the anomalistic period. Mean anomaly, for which comparison would not be so straightforward, is omitted from consideration.

6.1 Eccentricity, inclination, right ascension of the node and argument of perigee

To O(e) the relations between e, i, Ω, ω and $\bar{e}$, $\bar{i}$, $\bar{\Omega}$, $\bar{\omega}$ respectively are given by:-

$$e - \bar{e} = \frac{3}{8} J_2 \left(\frac{R}{a}\right)^2 e\,(2 - 3\sin^2 i) - \frac{1}{2}\,\frac{J_3}{J_2}\,\frac{R}{a} \sin i \sin \omega \quad ,$$

$$i - \bar{i} = -\frac{1}{8} J_2 \left(\frac{R}{a}\right)^2 \sin 2i\,(3 + 4e\cos\omega) + \frac{1}{2}\,\frac{J_3}{J_2}\,\frac{R}{a}\, e \cos i \sin \omega \quad ,$$

$$\Omega - \bar{\Omega} = \frac{1}{2} J_2 \left(\frac{R}{a}\right)^2 \cos i\,(3\omega + 3M_o + 4e\sin\omega) - \frac{1}{2}\,\frac{J_3}{J_2}\,\frac{R}{a}\, e \cot i \cos \omega$$

and

$$\omega - \bar{\omega} = -\frac{1}{4} J_2 \left(\frac{R}{a}\right)^2 \left\{(4 - 5\sin^2 i)(3\omega + 3M_o + 2e\sin\omega) - \frac{9}{4}\sin^2 i \sin 2\omega\right\} - \frac{1}{2}\,\frac{J_3}{J_2}\,\frac{R}{a}\, e^{-1} \sin i \cos \omega \quad .$$

The first term in the expression for $e - \bar{e}$ is negligible here since $2 - 3\sin^2 i = O(e)$; the second term represents the transformation made to the plot of NASA eccentricity in Fig.19.

6.2 Semi-major axis, mean motion and anomalistic period

The semi-major axis and the mean motion are directly related by Kepler's third law. Since - with observations of direction and not range - it is n rather than a which is measured, it is preferable to give the formula relating n and $\bar{n}$. There being no long-periodic term, the relation - to O(e) - is:-

$$n - \bar{n} = \frac{3}{4} J_2 \left(\frac{R}{a}\right)^2 n\,(2 - 3\sin^2 i) \ .$$

Although the true anomalistic period - the time from one passage through perigee to the next - is $2\pi/n$, it appears to be $2\pi/\bar{n}$ which is listed as 'anomalistic period' by NASA. Thus the latter is consistently half a second greater than the true anomalistic period.

Values listed in the Spacetrack bulletins do appear to be of the true anomalistic period and agree satisfactorily with values of $2\pi/n$ using Table 2.

6.3 Comparison of ephemerides

It does not matter that the definitions of R.A.E. and NASA elements are different, so long as the appropriate formulae are used for generating satellite positions in each case. A comparison has been made of satellite position computed from R.A.E. orbital elements with the ephemeris provided by the NASA 'Refined World Maps'. The period from node 600 to node 601 was chosen for the comparison since during this period the difference between e (NASA) and e (R.A.E.) was near its maximum (0.0008), as may be seen from Fig.19.

Fig.20 gives (lower curve) the difference between the R.A.E. and NASA computed satellite positions and (upper curve) the height component of this difference. It is seen that the height difference does not exceed 310 metres nor the total difference 800 metres. Since the R.A.E. elements are the more accurate, position agreement to this order is very satisfactory and confirms that both sets of orbital elements are being used correctly.

7 DISCUSSION

The accuracy required by the Ariel 2 experimenters has been achieved in the orbit provided. Had an order better accuracy been required, a careful analysis of the possible sources of error would have been necessary. At the expense of some complication, further improvements in the dynamic model of the orbit might have been made and the Pegasus computer programme improved. However, it is doubtful whether the Minitrack data themselves would have been accurate enough to warrant much refinement in the computing.

The average size of the residuals in angle is the 1 minute of arc that had been expected when the analysis was started. For a satellite at an average distance of about 1000 km this corresponds to an error in distance of about 300 m.

For observations with elevations as low as 60° the greatest source of error is probably the inadequate correction of ionospheric refraction. It has been estimated[10] that for an elevation angle of 50° the residual error after refraction correction is 2 minutes of arc so that for 60° it is still important.

Other errors in the data are not likely to be significant. Systematic errors which arise during the periods between successive calibrations of a Minitrack station - periods of some months in general - should not average more than ¼ minute of arc[1]. Errors in station survey should not exceed 100 m, leading to errors of the same order as calibration errors.

Turning to the orbital model, appreciable errors - say about 200 m - still arise from inadequate allowance for the earth's tesseral harmonics (see the Appendix), despite the empirical modification made. Errors from neglect of the zonal harmonics beyond J_4 are thought to be negligible. The effect of J_7, for example, is not important for a satellite of inclination $51^\circ.6$. The values of e_1 in Table 2, which are from theory, are in general agreement with values from differencing e, bearing in mind that the orbital model automatically allows for the main drag term in e - i.e., if e_1 is zero it is $a(1 - e)$ and not e which is held constant.

Errors from inadequate representation of the effects of atmospheric drag are less easy to estimate. They may be divided into short-periodic and long-periodic errors. The short-periodic errors arise because drag is concentrated at perigee and not, as the mean anomaly polynomial would suggest, uniformly spread over the orbit. But the errors are negligible, being less than a second of arc - in the case of Ariel 2 - for mean anomaly and so equivalent to at most a few seconds of arc in an observation.

The long-periodic errors in the representation of drag effects are more serious and it is these which are difficult to estimate. They arise because of fluctuations in the density of the atmosphere which are, on the different time scale involved, of short period. These fluctuations have been investigated by King-Hele and Quinn[11] who have plotted values of n_1, as in Fig.9, as far as 10th September, 1964. Geomagnetic index, when plotted for comparison with the parameter n_1, shows the same general behaviour but has very pronounced 'spikes'. It is clear, in fact, that the calculation of n_1, even at intervals as fine as 1¾ days, has smoothed the true orbital acceleration and thereby introduced error. If the required corrections to n_1 could be estimated and doubly integrated, then the correction to mean anomaly could be found as a function of time. From a rough inspection of the n_1 and geomagnetic index curves it seems likely that the error could reach, or exceed, a minute of arc in equivalent topocentric observation.

It is interesting, in the light of the last paragraph, to speculate that to obtain more accurate orbits for satellites like Ariel 2 it may be necessary to feed in values of geomagnetic index over the three-or-four-day period of an

orbit determination and to form terms contributing to the mean anomaly by a double numerical integration of these values of the index.

In addition to its application to the study of short-periodic fluctuations in atmospheric density the orbit of Ariel 2 has been used in the evaluation of the odd zonal harmonics of the earth's gravitational potential. King-Hele, Cook and Scott[12] have taken Ariel 2 as one of six satellites for this evaluation. They used the data for perigee radius, a (1 - e), rather than eccentricity, in order to remove drag effects as accurately as possible, and found that the corrected data were then as good as data for the other satellites in virtually drag-free orbits.

8 CONCLUSIONS

By use of an orbital model with eight parameters fitted from Minitrack observations, the orbit of Ariel 2 (1964-15A) has been successfully computed for a period of a year from its launch. The orbital parameters have been listed (in Table 2) for the time of every twenty-fifth passage through the ascending node.

Ephemerides calculated from the listed parameters should, in general, be accurate to about $\frac{1}{2}$ km, i.e. to the accuracy required by the experimenters. This estimate of accuracy is based on the view that the effective average error in a Minitrack observation is 1 minute of arc, a figure supported by the residuals in the observations after orbits have been fitted. The estimate has been confirmed by the comparison, over a short period, of ephemerides calculated from two sets of orbital parameters, corresponding to the epochs before and after the period in question.

Consideration has been given to the factors limiting the accuracy of the orbital determinations. It is concluded that the three main factors are: inadequacies in the correction of observations, other than at very high elevation, for ionospheric refraction; inadequacies in the representation of tesseral terms in the earth's gravitational field; and inadequacies in the representation of drag effects over periods of several days.

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Appendix

PERTURBATIONS OF ARIEL 2 DUE TO THE TESSERAL HARMONICS OF THE EARTH'S GRAVITATIONAL FIELD

During the first few orbital determinations, based on the standard dynamic model that was being used at the beginning of 1964, it was noticed that residuals from the Winkfield Minitrack station were consistently larger than residuals from the other stations. It was discovered moreover - using the original form of the orbit determination programme in which residuals in observation times as well as angular data were computed[2] - that the residuals were almost entirely along-track, equivalent to an error in the Winkfield clock. This error was oscillatory, with amplitude about $0^s.1$ and period just under (by about 10 minutes) half a day. It was realised that an error of this type could be caused by the neglect of the coefficient ($J_{2,2}$) of one of the tesseral harmonics of the earth's gravitational field, though it had earlier been thought that $J_{2,2}$ perturbations would not be detectable. The proof that the effect was in fact caused by gravitational perturbations - and that the Winkfield clock was blameless - was obtained by plotting time residuals for all the 185 observations which were obtained from the Minitrack network during the first 90 hours from launch. The residuals were plotted against the argument $\theta - \Omega$, where θ is sidereal time and Ω the right ascension of the satellite node.

Let the negative of the residual in an observation time (assuming the original form of the orbit determination programme) be denoted by δt. Then δt is the amount by which an observation time has to be increased before computing satellite position from the standard dynamic model. It was found that a good fit to the plot of 185 δt's was given by:-

$$\delta t = 0.088 \sin 2(\theta - \Omega - 18^\circ) \text{ sec} .$$

Now the formula for the along-track effect of $J_{2,2}$ is given[13], for a circular orbit, by:-

$$\delta L = \frac{9}{2} J_{2,2} \left(\frac{R}{p}\right)^2 \frac{n \sin^2 i}{\dot{\ell}} \sin 2(\theta - \Omega + \lambda_{2,2}) ,$$

where p, n and i are semi-latus rectum, mean motion and inclination respectively, R is the (mean) equatorial radius of the earth, $\lambda_{2,2}$ is the (east) longitude of one extremity of the major axis of the earth's equator, assumed elliptical, and $\dot{\ell}$ is $\dot{\theta} - \dot{\Omega}$. Taking $\delta L = n\,\delta t$ and substituting the values of p, n, i and $\dot{\Omega}$ for the Ariel 2 orbit, the observed δt fit may be accounted for by a $J_{2,2}$

perturbation for which $J_{2,2} = 3.0 \times 10^{-6}$ and $\lambda_{2,2} = -18^\circ$. The orbit determination programme has consequently been modified by arranging that the time of an observation is increased by δt where

$$\delta t = \frac{9}{2} J_{2,2} \left(\frac{R}{p}\right)^2 \frac{\sin^2 i}{\ell} \sin 2(\theta - \Omega + \lambda_{2,2}) ,$$

$J_{2,2}$ and $\lambda_{2,2}$ having the values above.

Now the values of $J_{2,2}$ and $\lambda_{2,2}$ obtained by Guier and Newton[14], with which other values are in fair agreement, are 1.72×10^{-6} and $-13^\circ.4$ respectively, so that at first sight the correction is almost double what can be justified. The natural explanation is that the fitted $J_{2,2}$ accounts not only for the pure $J_{2,2}$ but also for (the along-track effects of) $J_{4,2}$, $J_{6,2}$, $J_{8,2}$ etc. However, this explanation has not been satisfactorily borne out on evaluating these effects as far as $J_{8,2}$, using values from Ref.14. The pure $J_{2,2}$ effect and the $J_{4,2}$, $J_{6,2}$ and $J_{8,2}$ effects are then given, respectively, by:-

$$\delta t = 0.051 \sin 2(\theta - \Omega - 13^\circ) ,$$

$$\delta t = 0.017 \sin 2(\theta - \Omega - 67^\circ) ,$$

$$\delta t = 0.004 \sin 2(\theta - \Omega + 22^\circ)$$

and

$$\delta t = 0.002 \sin 2(\theta - \Omega - 11^\circ) ;$$

the formulae for the quantities $L_{2,2}$ and $L_{4,2}$ of Ref.13 have been used here, and similar formulae developed for $L_{6,2}$ and $L_{8,2}$. On combining, the total effect is given by:-

$$\delta t = 0.051 \sin 2(\theta - \Omega - 21^\circ)$$

so that the amplitude is the same as for the pure $J_{2,2}$ effect though the phase has changed. Values for $J_{10,2}$ etc. are not available but there seems to be little hope of increasing the amplitude to the observed 0.088 sec through their agency.

The efficacy of the empirical programme modification may be appreciated when it is stated that the mean value of the observation residuals is halved. Although the modification was based on the analysis at the early nodes it was

tested also at later nodes; at node 3250, for example, the mean (rms) residual was 2'.80 without the modification, but 1'.46 with it.

It is recalled that the programme modification accounts for only that component of the pertubation due to tesseral harmonics which manifests itself as an apparent along-track error in satellite position with period about half a day. There are also components in the two directions perpendicular to the along-track direction. The perturbation which is cross-track but within the orbital plane contains eccentricity as a factor and may be neglected, but the perturbation perpendicular to the orbital plane is significant. Its effect is less noticeable than that of the along-track component, partly because it has a short-periodic factor superimposed on the half-day period. Its removal, however, might be expected to contribute to a further improvement in orbital fits.

For a complete representation of the effect of any particular tesseral harmonic the natural procedure would be to introduce the perturbations of the six orbital elements. With the normal notation[13] (but with σ for the element sometimes denoted by χ^*) there would be no perturbation in a, while perturbations in ω and σ could, for a near-circular orbit, be combined; thus expressions for δe, δi, $\delta\Omega$ and $\delta\omega + \delta\sigma$ would be required instead of merely the expression for δL ($= \delta\omega + \delta\sigma + \delta\Omega \cos i$).

Attempts have been made to represent the complete perturbations of Ariel 2 due to $J_{2,2}$ and $J_{4,2}$ by taking either the values of Ref.14 or else the empirical values for the along-track perturbation only, viz. $J_{2,2} = 3.0 \times 10^{-6}$, $\lambda_{2,2} = -18^{\circ}$ and $J_{4,2} = 0$. Both attempts were unsuccessful - residuals became larger not smaller - and perhaps this is not surprising. Since it was necessary to take an empirical $J_{2,2}$, based on residual fitting, to represent the along-track perturbation alone successfully, it would presumably be necessary to take a new empirical value to represent the complete perturbation. But the method of residual fitting would not then be so obvious; since the revised orbital model - with the empirical along-track correction - gave the Ariel 2 orbit to the required accuracy, the question of further improvement has not been closely studied.

The perturbations due to $J_{n,s}$ with $s \neq 2$ would, of course, have to be considered in a full analysis. The tables below give order-of-magnitude estimates of the amplitudes of the perturbations from $J_{n,s}$ as far as $J_{4,4}$ using values from Ref.14, the effects on the elements being represented in metres. Lines of the tables for which the perturbations include eccentricity ($e = 0.07$) as a factor are indicated by an asterisk.

δe

n \ s	1	2	3	4
2*		[illegible]		
3	130	45	25	
4*	4	2	1	1

δi

n \ s	1	2	3	4
2		160		
3*	20	10	5	
4	12	30	50	7

$\delta\Omega$

n \ s	1	2	3	4
2		130		
3*	25	10	3	
4	170	40	20	5

$\delta\omega + \delta\sigma$

n \ s	1	2	3	4
2		300		
3*	120	25	10	
4	45	150	110	10

Table 1

MINITRACK STATIONS OBSERVING ARIEL 2

Station location	Latitude	Longitude	Height (metres)	No. of observations
Blossom Point, Maryland, USA	38.43053 N	77.08629 W	-1	252
East Grand Forks, Minnesota, USA	48.02256 N	97.01082 W	254	760
Fort Myers, Florida, USA	26.54827 N	81.86539 W	7	225
Goldstone Lake, California, USA	35.33016 N	116.89977 W	937	202
Hartebeeshoek, Johannesburg, SA	25.88361 S	27.70791 E	1571	216
Lima, Peru	11.77635 S	77.15024 W	2	210
Quito, Ecuador	00.62237 S	78.57900 W	3548	140
Santiago, Chile	33.14896 S	70.66865 W	636	174
St. Johns, Newfoundland	47.74137 N	52.72036 W	106	698
Winkfield, England	51.44595 N	0.69623 W	91	649
Woomera, Australia	31.39167 S	136.86972 E	118	189

Latitude and longitude are referenced to the Fischer ellipsoid: a spheroid of semi-major axis 6378.166 km and flattening 1/298.3. Heights are measured upwards from this spheroid.

Table 2

ORBITAL PARAMETERS OF ARIEL 2

Node	Date 1964	Time h m s	a	e	$10^4 e_1$	a(1-e)	i	100 i_1	Ω	100 $Ω_1$	ω	$ω_1$	M	n	n_1	$10^{-2} n_2$	N	D	ε	MJD
25	MAR 29	11 06 46.53 2	7201.0102 7	0.074629 6	-36	6663.61	51.6450 4	4	-137.2597 9	30	143.137 5	-2.83	-137.769	511465.06	2316 12	210 13	44	3.5	1.5	38483.4630385
50	MAR 31	05 18 51.65 2	7200.5681 7	0.074518 7	-38	6664.00	51.6458 4	1	-144.4483 10	30	148.442 5	-2.58	-143.751	511512.17	2825 7	20 12	35	3.2	1.1	38485.2214311
75	APR 1	23 30 41.33 3	7200.0919 9	0.074388 13	-39	6664.49	51.6461 5	0	-151.6363 13	28	153.757 7	-2.36	-149.791	511562.91	3207 9	248 14	45	3.8	1.4	38486.9/96450
100	APR 3	17 42 13.81 3	7199.5452 8	0.074254 9	-40	6664.95	51.6456 4	0	-158.8259 10	27	159.067 6	-2.11	-155.862	511621.18	2954 9	-160 14	37	3.2	1.0	38488.7376598
125	APR 5	11 53 30.16 3	7199.0946 7	0.074128 9	-40	6665.44	51.6460 5	2	-166.0100 5	26	164.394 7	-1.81	-161.981	511669.22	2624 6	-63 8	46	4.1	1.5	38490.4954880
150	APR 7	06 04 32.40 3	7198.6802 8	0.074010 6	-41	6665.91	51.6460 5	4	-173.2018 6	25	169.728 7	-1.42	-168.129	511713.40	2439 7	-30 12	36	3.1	1.3	38492.2531528
175	APR 9	00 15 21.59 3	7198.2829 8	0.073899 7	-42	6666.34	51.6468 6	3	179.6053 7	25	175.073 7	-1.01	-174.303	511755.76	2311 6	-95 10	40	3.9	1.7	38494.0106666
200	APR 10	18 25 58.68 4	7197.9319 12	0.073781 8	-42	6666.86	51.6473 8	0	172.4118 10	24	180.428 7	-0.65	-180.495	511793.19	1998 13	-34 21	30	3.5	1.8	38495.7680403
225	APR 12	12 36 25.30 3	7197.6090 10	0.073637 7	-42	6667.60	51.6477 6	-3	165.2181 9	21	185.788 6	-0.41	-186.688	511827.64	1850 6	-78 10	41	4.1	1.5	38497.5252928
250	APR 14	06 46 42.54 2	7197.3301 9	0.073516 6	-40	6668.21	51.6465 5	-2	158.0274 10	19	191.140 6	-0.24	-192.860	511857.39	1570 9	-43 15	36	3.5	1.0	38499.2824367
275	APR 16	00 56 51.88 2	7197.0828 7	0.073410 6	-39	6668.74	51.6468 4	1	150.8343 11	18	196.503 7	0.00	-199.025	511883.78	1557 7	49 9	40	4.1	1.0	38501.0394893
300	APR 17	19 06 53.59 3	7196.8140 11	0.073293 7	-37	6669.34	51.6476 5	3	143.6415 14	19	201.870 10	0.36	-205.165	511912.45	1614 8	-31 16	32	3.4	1.0	38502.7964536
325	APR 19	13 16 47.46 3	7196.5531 8	0.073191 7	-35	6669.83	51.6469 5	2	136.4496 16	20	207.230 11	0.75	-211.263	511940.29	1517 6	-43 7	35	4.1	1.0	38504.5533271
350	APR 21	07 26 34.16 4	7196.3243 12	0.073064 10	-34	6670.53	51.6461 4	-1	129.2553 14	18	212.619 6	1.05	-217.348	511964.71	1216 14	-67 28	31	2.9	0.9	38506.3101175
375	APR 23	01 36 15.26 3	7196.1476 6	0.072966 9	-32	6671.07	51.6453 4	-2	122.0602 12	15	218.001 5	1.22	-223.378	511983.56	970 5	-49 6	43	4.2	0.8	38508.0668433
400	APR 24	19 45 52.20 3	7196.0019 13	0.072889 8	-30	6671.49	51.6445 5	0	114.8657 10	14	223.416 5	1.34	-229.391	511999.11	885 13	25 24	33	3.0	0.9	38509.8235208
425	APR 26	13 55 25.37 2	7195.8376 7	0.072825 7	-26	6671.80	51.6459 5	2	107.6682 8	14	228.817 4	1.47	-235.327	512016.64	1127 6	68 7	46	4.7	1.1	38511.5801547
450	APR 28	08 04 53.95 2	7195.6274 13	0.072732 6	-22	6672.28	51.6460 5	4	100.4777 8	16	234.209 5	1.63	-241.187	512039.08	1395 14	-50 26	31	3.1	0.9	38513.3367355
475	APR 30	02 14 16.65 2	7195.4266 9	0.072672 7	-18	6672.52	51.6480 6	3	93.2805 9	18	239.605 5	1.83	-246.987	512060.51	1082 8	-50 9	38	4.2	1.2	38515.0932482
500	MAY 1	20 23 34.92 3	7195.2569 12	0.072616 8	-15	6672.84	51.6493 5	1	86.0833 8	18	245.022 7	2.03	-252.737	512078.63	1037 14	-20 22	26	3.1	0.9	38516.8497097
525	MAY 3	14 32 49.37 3	7195.1047 9	0.072541 7	-12	6673.16	51.6491 4	0	78.8966 9	16	250.459 6	2.20	-258.431	512094.88	787 7	-54 12	41	3.6	0.8	38518.6061270
550	MAY 5	08 42 00.69 2	7194.9865 12	0.072505 7	-09	6673.31	51.6486 5	0	71.6988 9	15	255.871 6	2.29	-264.030	512107.49	749 16	41 28	35	2.9	0.8	38520.3625079
575	MAY 7	02 51 09.53 3	7194.8685 11	0.072471 8	-07	6673.45	51.6486 8	1	64.5054 10	14	261.279 8	2.28	-269.549	512120.10	663 8	-22 11	44	4.1	1.3	38522.1188603
600	MAY 8	21 00 16.14 2	7194.7657 15	0.072429 6	-03	6673.66	51.6503 8	3	57.3088 9	16	266.697 6	2.20	-275.002	512131.08	595 15	7 29	32	3.0	1.2	38523.8751869
625	MAY 10	15 09 20.85 2	7194.6556 11	0.072405 8	02	6673.73	51.6520 9	2	50.1135 11	19	272.122 7	2.10	-280.389	512142.83	892 11	134 13	41	4.0	1.9	38525.6314913
650	MAY 12	09 18 22.06 2	7194.4792 12	0.072402 8	07	6673.58	51.6503 8	-1	42.9211 9	19	277.542 6	2.08	-285.702	512161.66	781 17	-201 26	33	3.0	1.4	38527.3877554
675	MAY 14	03 27 20.00 2	7194.3744 10	0.072405 8	11	6673.47	51.6496 7	-3	35.7253 9	18	282.959 5	2.14	-290.941	512172.86	751 9	98 17	40	3.3	1.4	38529.1439815
700	MAY 15	21 36 15.16 1	7194.2291 10	0.072405 11	14	6673.33	51.6492 5	-2	28.5299 8	16	288.378 4	2.19	-296.113	512188.37	880 15	-34 30	35	2.9	1.0	38530.9001755
725	MAY 17	15 45 06.83 3	7194.0940 14	0.072413 14	16	6673.15	51.6475 7	0	21.3311 16	18	293.792 7	2.12	-301.216	512202.80	778 14	-26 18	35	3.7	1.4	38532.6563290
750	MAY 19	09 53 55.59 3	7193.9794 13	0.072431 9	18	6672.91	51.6488 5	-1	14.1364 17	20	299.223 10	1.93	-306.274	512215.04	608 13	-18 22	21	3.1	0.7	38534.4124489
775	MAY 21	04 02 42.08 5	7193.8771 19	0.072459 8	22	6672.62	51.6479 6	-1	6.9410 16	22	304.649 14	1.66	-311.270	512225.96	615 16	-40 28	21	3.1	0.9	38536.1685426
800	MAY 22	22 11 26.13 3	7193.7749 13	0.072484 5	25	6672.34	51.6485 6	-3	-0.2569 14	22	310.038 13	1.41	-316.178	512236.88	685 19	2 24	24	3.1	0.9	38537.9246079
825	MAY 24	16 20 07.53 3	7193.6525 10	0.072513 6	29	6672.02	51.6477 5	-4	-7.4546 13	21	315.432 11	1.20	-321.043	512249.95	901 19	53 21	23	3.1	0.8	38539.6806427
850	MAY 26	10 28 45.15 2	7193.4928 9	0.072583 8	33	6671.37	51.6485 4	-2	-14.6466 8	21	320.822 6	1.03	-325.861	512267.01	940 7	-28 9	21	4.0	0.8	38541.4366336
875	MAY 28	4 37 18.43 1	7193.3460 9	0.072635 7	36	6670.86	51.6477 4	-1	-21.8470 6	22	326.198 4	0.88	-330.626	512282.70	879 9	-23 15	19	3.3	0.8	38543.1925744

Table 2 (Contd)

ORBITAL PARAMETERS OF ARIEL 2

Node	Date 1964	Time h m s	a	e	$10^4 e_1$	a(1−e)	i	100 i_1	Ω	100 $Ω_1$	ω	$ω_1$	M	n	n_1	$10^{-2} n_2$	N	D	ε	MJD
900	MAY 29	22 45 47.77 1	7193.2086 6	0.072687 10	38	6670.36	51.6473 4	-1	-29.0490 5	25	331.581 4	0.70	-335.362	512297.37	817 8	-7 9	19	3.4	0.7	38544.9484696
925	MAY 31	16 54 13.58 5	7193.0804 14	0.072802 18	39	6669.41	51.6479 6	-2	-36.2444 12	27	336.976 10	0.12	-340.081	512311.07	746 25	-37 27	18	3.0	1.0	38546.7043238
950	JUN 2	11 02 55.73 4	7192.9657 20	0.072821 20	39	6669.17	51.6466 8	-3	-43.4444 14	28	342.349 11	0.12	-344.753	512323.32	641 27	-46 47	26	3.5	1.4	38548.4601358
975	JUN 4	5 10 54.79 3	7192.8584 8	0.072890 9	40	6668.57	51.6459 3	-4	-50.6454 7	27	347.716 6	-0.28	-349.403	512334.79	685 6	+34 12	30	3.4	0.7	38550.2159119
1000	JUN 5	23 19 10.53 3	7192.7439 10	0.072948 12	42	6668.05	51.6444 4	-2	-57.8434 11	26	353.086 6	-0.68	-354.040	512347.02	660 7	-24 14	26	3.3	0.8	38551.9716497
1025	JUN 7	17 27 23.10 2	7192.6443 10	0.073076 7	44	6667.03	51.6437 4	0	-65.0395 9	28	358.441 7	-0.98	-358.657	512357.66	630 11	+64 17	26	3.0	0.9	38553.7273507
1050	JUN 9	11 35 32.18 2	7192.5216 12	0.073127 9	45	6666.55	51.6433 6	0	-72.2431 12	31	3.797 8	-1.21	-3.271	512370.77	1095 12	+266 17	23	3.6	1.3	38555.4830113
1075	JUN 11	5 43 35.46 4	7192.2695 11	0.073156 17	45	6666.11	51.6425 4	-3	-79.4453 14	35	9.157 7	-1.46	-7.892	512397.71	1573 10	-116 19	25	3.1	1.0	38557.2386049
1100	JUN 12	23 51 30.70 2	7192.0487 6	0.073234 9	43	6665.35	51.6418 3	-5	-86.6458 9	35	14.530 5	-1.80	-12.532	512421.31	1133 5	-110 10	37	3.5	1.0	38558.9941053
1125	JUN 14	17 59 19.65 2	7191.8860 9	0.073286 12	41	6664.82	51.6415 5	-4	-93.8491 13	34	19.876 7	-2.20	-17.162	512438.70	964 8	-3 14	34	3.2	1.5	38560.7495330
1150	JUN 16	12 07 03.31 2	7191.7379 6	0.073340 9	40	6664.30	51.6420 3	-1	-101.0504 9	33	25.212 5	-2.57	-21.803	512454.53	939 8	20 12	29	2.9	0.9	38562.5048994
1175	JUN 18	6 14 42.08 2	7191.6025 4	0.073376 7	39	6663.91	51.6426 3	0	-108.2541 8	35	30.544 4	-2.89	-26.467	512469.00	786 6	3 8	33	3.7	0.9	38564.2602093
1200	JUN 20	0 22 16.21 2	7191.4648 7	0.073433 5	37	6663.37	51.6437 3	0	-115.4568 9	37	35.845 6	-3.15	-31.131	512483.71	990 6	92 12	28	3.1	1.1	38566.0154654
1225	JUN 21	18 29 44.66 1	7191.2875 6	0.073477 4	35	6662.89	51.6432 3	-2	-122.6598 8	38	41.134 7	-3.40	-35.819	512502.67	1013 5	-71 8	24	3.3	0.9	38567.7706558
1250	JUN 23	12 37 07.16 1	7191.1317 5	0.073498 5	32	6662.60	51.6423 3	-4	-129.8613 6	39	46.460 6	-3.63	-40.582	512519.32	963 6	-17 9	27	3.0	0.9	38569.5257773
1275	JUN 25	6 44 23.96 2	7190.9768 7	0.073527 6	28	6662.25	51.6417 4	-4	-137.0669 7	38	51.779 5	-3.86	-45.383	512535.88	1016 9	73 16	26	3.0	1.0	38571.2808329
1300	JUN 27	0 51 34.69 2	7190.8043 7	0.073558 7	24	6661.86	51.6403 4	-2	-144.2716 8	37	57.081 5	-4.07	-50.216	512554.33	986 8	-33 13	26	3.2	1.1	38573.0358182
1325	JUN 28	18 58 39.32 1	7190.6436 7	0.073593 5	21	6661.46	51.6402 3	1	-151.4784 7	37	62.374 4	-4.28	-55.095	512571.51	1007 5	20 10	29	3.4	1.0	38574.7907329
1350	JUN 30	13 05 37.71 2	7190.4791 11	0.073603 7	18	6661.24	51.6411 4	3	-158.6855 11	38	67.668 8	-4.46	-60.033	512589.11	910 8	-66 17	24	3.1	1.6	38576.5455753
1375	JUL 2	7 12 30.39 2	7190.3463 10	0.073618 8	14	6661.01	51.6396 5	1	-165.8891 14	39	72.988 9	-4.59	-65.057	512603.30	840 10	70 16	17	3.5	1.5	38578.3003517
1400	JUL 4	1 19 17.53 4	7190.1833 12	0.073627 9	11	6660.79	51.6394 5	-1	-173.0946 15	38	78.287 13	-4.66	-70.124	512620.74	1158 14	55 28	17	3.2	1.7	38580.0550640
1425	JUL 5	19 25 57.69 2	7190.0006 6	0.073628 6	06	6660.62	51.6395 3	-2	179.6976 10	38	83.614 7	-4.72	-75.288	512640.27	1012 6	-35 10	30	3.2	1.4	38581.8096955
1450	JUL 7	13 32 31.33 2	7189.8325 7	0.073605 6	01	6660.62	51.6397 3	2	172.4915 10	36	88.925 7	-4.62	-80.508	512658.25	1207 5	129 10	36	3.3	1.6	38583.5642515
1475	JUL 9	7 38 57.54 2	7189.6109 6	0.073580 5	-04	6660.60	51.6404 3	5	165.2832 9	37	94.222 5	-4.70	-85.786	512681.95	1304 6	-44 10	30	3.3	1.3	38585.3187215
1500	JUL 11	1 45 15.74 2	7189.4126 4	0.073534 5	-08	6660.75	51.6422 4	5	158.0770 7	38	99.513 5	-4.71	-91.134	512703.16	1096 9	-49 5	26	4.4	1.2	38587.0730988
1525	JUL 12	19 51 26.99 2	7189.2479 4	0.073502 4	-11	6660.82	51.6431 3	3	150.8666 7	38	104.812 6	-4.64	-96.560	512720.78	959 4	-9 5	24	3.1	1.1	38588.8273957
1550	JUL 14	13 57 32.04 3	7189.0938 11	0.073466 6	-15	6660.94	51.6439 4	0	143.6602 8	36	110.119 11	-4.50	-102.068	512737.26	881 8	-47 16	20	3.1	1.4	38590.5816208
1575	JUL 16	8 03 31.25 3	7188.9585 9	0.073412 5	-19	6661.20	51.6443 5	1	136.4517 9	33	115.427 10	-4.31	-107.652	512751.74	850 9	54 13	32	3.2	1.6	38592.3357784
1600	JUL 18	2 09 24.80 2	7188.7909 8	0.073356 6	-23	6661.45	51.6441 4	3	129.2423 10	33	120.724 6	-4.11	-113.294	512769.67	1230 10	11 15	36	3.1	1.4	38594.0898704
1625	JUL 19	20 15 11.00 2	7188.5878 6	0.073292 6	-27	6661.72	51.6447 3	4	122.0320 8	33	126.032 4	-3.90	-119.018	512791.41	1190 10	-37 14	36	2.9	1.2	38595.8438773
1650	JUL 21	14 20 50.01 3	7188.4092 5	0.073226 11	-30	6662.03	51.6446 4	5	114.8229 12	34	131.342 7	-3.70	-124.809	512810.52	1009 9	-44 7	36	3.1	1.4	38597.5978010
1675	JUL 23	8 26 22.71 3	7188.2445 8	0.073148 9	-34	6662.44	51.6450 4	3	107.6157 13	33	136.647 9	-3.50	-130.657	512828.14	1000 6	-16 11	32	3.3	1.4	38599.3516517
1700	JUL 25	2 31 49.16 3	7188.0841 7	0.073060 4	-36	6662.92	51.6452 4	1	100.4063 10	31	141.977 7	-3.26	-136.590	512845.31	994 9	32 14	34	3.1	1.3	38601.1054301
1725	JUL 26	20 37 09.54 3	7187.9143 7	0.072984 4	-37	6663.31	51.6458 4	0	93.1967 7	30	147.310 7	-2.99	-142.578	512863.48	1017 9	-15 15	41	3.4	1.4	38602.8591382
1750	JUL 28	14 42 23.86 3	7187.7482 9	0.072904 4	-38	6663.73	51.6459 4	1	85.9846 8	25	152.640 7	-2.64	-148.606	512881.25	1058 11	31 15	43	3.1	1.6	38604.6127762

Table 2 (Contd)

ORBITAL PARAMETERS OF ARIEL 2

Node	Date 1964	Time h m s	a	e	$10^4 e_1$	a(1−e)	i	$100\, i_1$	Ω	$100\, \Omega_1$	ω	ω_1	M	n	n_1	$10^{-2} n_2$	N	D	ε	MJD
1775	JUL 30	8 47 31.92 3	7187.5502 7	0.072804 5	−39	6664.27	51.6466 4	3	73.7742 8	25	157.977 6	−2.24	−154.693	[illegible]02.45	1380 10	79 14	35	3.1	1.2	38606.3663417
1800	AUG 1	2 52 32.40 3	7187.3182 9	0.072704 7	−41	6664.77	51.6473 4	4	71.5616 8	26	163.311 6	−1.84	−160.787	512927.28	1322 7	−40 14	31	3.3	1.1	38608.1198195
1825	AUG 2	20 57 25.65 3	7187.1085 6	0.072599 8	−43	6665.33	51.6477 5	3	64.3562 9	26	168.657 7	−1.52	−166.929	512949.73	1294 12	−6 3	31	2.9	1.5	38609.8732135
1850	AUG 4	15 2 11.69 3	7186.8681 10	0.072486 6	−44	6665.92	51.6478 6	0	57.1421 9	23	174.004 6	−1.29	−173.087	512975.47	1712 13	90 17	33	3.4	1.5	38611.6265242
1875	AUG 6	9 6 48.84 2	7186.5800 11	0.072355 9	−43	6666.60	51.6482 5	0	49.9282 12	20	179.372 7	−1.08	−179.277	513006.32	1594 12	−84 20	28	3.1	1.1	38613.3797319
1900	AUG 8	3 11 17.64 3	7186.3247 10	0.072252 6	−41	6667.10	51.6484 4	2	42.7189 9	18	184.726 5	−0.76	−185.447	513033.65	1545 8	−52 14	34	3.4	1.0	38615.1328431
1925	AUG 9	21 15 38.52 4	7186.0804 13	0.072158 11	−39	6667.55	51.6483 5	4	35.5043 11	19	190.107 8	−0.36	−191.640	513059.82	1472 11	−28 24	30	3.1	1.2	38616.8858625
1950	AUG 11	15 19 52.04 3	7185.8260 8	0.072046 6	−38	6668.12	51.6484 5	3	28.2932 8	20	195.506 6	0.06	−197.834	513087.06	1729 16	37 18	30	3.4	1.2	38618.6387967
1975	AUG 13	9 23 57.18 3	7185.5351 10	0.071937 6	−37	6668.63	51.6487 5	1	21.0770 8	20	200.887 6	0.42	−203.981	513118.22	1698 11	−66 19	36	3.1	1.4	38620.3916340
2000	AUG 15	3 27 54.24 3	7185.2791 14	0.071828 8	−36	6669.17	51.6493 6	0	13.8654 12	19	206.254 8	0.69	−210.078	513145.64	1502 12	−23 23	37	3.2	1.5	38622.1443778
2025	AUG 16	21 31 44.10 2	7185.0353 8	0.071722 8	−35	6669.71	51.6501 5	0	6.6521 9	17	211.662 6	0.89	−216.181	513171.76	1496 10	8 11	36	3.8	1.1	38623.8970382
2050	AUG 18	15 35 27.05 2	7184.7868 7	0.071604 5	−33	6670.33	51.6509 5	0	−0.5631 7	15	217.053 5	1.06	−222.216	513198.38	1483 10	−39 11	24	3.5	0.8	38625.6496186
2075	AUG 20	9 39 3.13 2	7184.5490 19	0.071515 6	−30	6670.75	51.6494 8	0	−7.7745 9	15	222.450 7	1.43	−228.206	513223.86	1387 25	−63 63	19	2.4	1.1	38627.4021195
2100	AUG 22	3 42 32.91 6	7184.3300 29	0.071441 14	−25	6671.07	51.6498 17	1	−14.9886 13	18	227.852 11	1.43	−234.144	513247.33	1273 22	−60 41	13	3.2	1.7	38629.1545476
2125	AUG 23	21 45 57.15 7	7184.1231 14	0.071327 23	−21	6671.70	51.6503 9	−1	−22.2014 8	19	233.306 11	1.67	−240.071	513269.51	1249 21	−94 27	9	2.9	0.8	38630.9069115
2150	AUG 25	15 49 16.17 4	7183.9376 16	0.071268 10	−18	6671.95	51.6488 8	−3	−29.4150 11	17	238.724 6	1.93	−245.899	513289.38	1169 41	−18 45	21	2.7	1.1	38632.6592149
2175	AUG 27	9 52 30.31 2	7183.7370 15	0.071225 7	−16	6672.08	51.6496 5	−3	−36.6305 7	15	244.141 4	2.14	−251.655	513310.89	1228 16	−117 40	39	2.6	1.2	38634.4114619
2200	AUG 29	3 55 39.52 2	7183.5601 11	0.071167 6	−14	6672.33	51.6498 5	−1	−43.8492 8	14	249.569 4	2.25	−257.350	513329.84	973 14	−71 21	41	3.1	1.2	38636.1636518
2225	AUG 30	21 58 44.93 2	7183.4090 9	0.071122 6	−11	6672.51	51.6502 6	0	−51.0654 8	15	255.008 7	2.25	−262.984	513346.04	1001 12	32 13	39	3.8	1.1	38637.9157978
2250	SEP 1	16 1 46.57 3	7183.2311 11	0.071053 7	−7	6672.84	51.6491 6	−2	−58.2771 7	18	260.456 5	2.21	−268.550	513365.11	1152 12	−6 16	35	3.3	1.1	38639.6679002
2275	SEP 3	10 4 43.43 2	7183.0474 9	0.071021 6	−2	6672.90	51.6488 7	−5	−65.4944 7	18	265.889 5	2.21	−274.031	513384.81	1126 13	6 15	34	3.5	1.3	38641.4199471
2300	SEP 5	4 7 35.74 2	7182.8645 11	0.071003 6	2	6672.86	51.6477 8	−5	−72.7085 10	17	271.335 6	2.28	−279.452	513404.42	1151 18	47 24	22	3.1	1.1	38643.1719414
2325	SEP 6	22 10 23.33 2	7182.6702 14	0.070985 7	6	6672.81	51.6468 7	−3	−79.9243 9	15	276.774 6	2.33	−284.795	513425.25	1340 12	152 22	17	3.2	0.8	38644.9238811
2350	SEP 8	16 13 5.07 5	7182.4167 15	0.070976 12	9	6672.64	51.6459 7	−1	−87.1415 9	15	282.215 13	2.35	−290.069	513452.43	1663 11	67 21	17	3.4	0.8	38646.6757532
2375	SEP 10	10 15 39.35 2	7182.1427 9	0.070967 5	12	6672.45	51.6464 6	0	−94.3605 8	17	287.634 5	2.28	−295.255	513481.82	1523 16	−82 18	17	3.1	0.7	38648.4275388
2400	SEP 12	4 18 6.85 2	7181.9114 19	0.070956 8	15	6672.31	51.6463 8	−1	−101.5785 8	19	293.072 7	2.15	−300.395	513506.62	1322 23	−100 47	19	2.7	1.1	38650.1792459
2425	SEP 13	22 20 28.49 2	7181.7089 13	0.070971 7	18	6672.02	51.6452 6	−3	−108.7976 6	21	298.501 5	1.97	−305.466	513528.34	1268 12	25 22	25	3.1	0.9	38651.9308853
2450	SEP 15	16 22 44.66 3	7181.5014 12	0.070982 6	22	6671.74	51.6460 6	−3	−116.0160 8	21	303.951 9	1.78	−310.499	513550.60	1262 47	2 49	19	2.4	0.7	38653.6824614
2475	SEP 17	10 24 54.82 1	7181.2539 8	0.071000 7	26	6671.38	51.6460 6	−2	−123.2337 7	21	309.387 5	1.60	−315.466	513577.15	1619 13	−50 15	29	3.2	1.3	38655.4339679
2500	SEP 19	4 26 57.52 2	7181.0087 11	0.071037 9	29	6670.89	51.6449 6	0	−130.4538 8	21	314.799 5	1.43	−320.363	513603.45	1394 12	−15 19	25	3.1	1.3	38657.1853879
2525	SEP 20	22 28 53.71 2	7180.7897 12	0.071064 9	31	6670.49	51.6443 6	2	−137.6739 12	23	320.218 7	1.28	−325.221	513626.95	1394 22	111 26	20	2.9	1.0	38658.9367327
2550	SEP 22	16 30 43.26 2	7180.5255 9	0.071084 6	33	6670.11	51.6452 5	1	−144.8924 9	25	325.641 6	1.10	−330.042	513655.29	1772 9	44 13	30	3.5	1.1	38660.6880006
2575	SEP 24	10 32 24.18 1	7180.2323 7	0.071134 4	35	6669.47	51.6452 4	−2	−152.1134 5	27	331.032 3	0.84	−334.802	513686.76	1722 6	−15 12	42	3.4	1.0	38662.4391687
2600	SEP 26	4 33 56.69 1	7179.9624 6	0.071187 5	37	6668.84	51.6442 4	−5	−159.3363 4	28	336.438 3	0.46	−339.545	513715.72	1589 8	−14 11	39	3.2	1.0	38664.1902394
2625	SEP 27	22 35 21.36 1	7179.7075 17	0.071221 8	39	6668.36	51.6437 5	−4	−166.5605 7	27	341.842 5	0.07	−344.262	513743.07	1811 13	322 46	30	2.5	1.0	38665.9412194

Table 2 (Contd)

ORBITAL PARAMETERS OF ARIEL 2

Node	Date 1964	Time h m s	a	e	$10^4 e_1$	a(1−e)	i	100 i_1	Ω	100 $Ω_1$	ω	$ω_1$	M	n	n_1	$10^{-2} n_2$	N	D	ε	MJD
2650	SEP 29	16 36 36.93 2	7179.3550 8	0.071265 7	41	6667.72	51.6434 5	0	-173.7817 9	28	347.251 7	-0.23	-348.963	513780.92	2157 15	-28 12	32	3.9	1.3	38667.6920941
2675	OCT 1	10 37 41.48 1	7179.0104 5	0.071302 5	42	6667.13	51.6427 4	2	178.9925 6	31	352.631 5	-0.43	-353.625	513817.91	2101 7	-32 14	29	3.4	0.9	38669.4428412
2700	OCT 3	4 38 35.45 2	7178.6798 10	0.071339 9	23	6666.56	51.6414 4	1	171.7671 11	39	358.020 6	-0.68	-358.288	513853.40	2079 10	142 17	24	3.1	0.9	38671.1934658
2725	OCT 4	22 39 18.59 3	7178.2934 9	0.071350 7	42	6666.12	51.6416 4	-4	164.5469 12	32	3.410 9	-1.04	-2.948	513894.89	2608 17	-35 20	25	2.9	1.1	38672.9439651
2750	OCT 6	16 39 48.52 1	7177.8920 6	0.071389 3	42	6665.47	51.6419 3	-5	157.3204 5	32	8.760 5	-1.48	-7.577	513938.00	2196 13	9 20	37	3.1	0.9	38674.6943116
2775	OCT 8	10 40 6.59 1	7177.5405 5	0.071426 4	42	6664.88	51.6411 3	-4	150.0922 4	32	14.122 4	-1.91	-12.225	513975.75	2152 5	27 8	49	3.4	1.0	38676.4445208
2800	OCT 10	4 40 13.27 1	7177.1817 7	0.071473 5	41	6664.21	51.6408 4	-1	142.8626 6	33	19.464 6	-2.26	-16.868	514014.30	2099 5	-117 10	38	3.3	1.2	38678.1945980
2825	OCT 11	22 40 8.90 2	7176.8724 17	0.071510 6	40	6663.65	51.6411 5	0	135.6353 9	35	24.799 9	-2.54	-21.523	514047.53	1887 9	84 26	26	3.1	1.5	38679.9445474
2850	OCT 13	16 39 54.24 3	7176.5419 18	0.071541 6	39	6663.12	51.6394 6	0	128.4073 11	36	30.172 10	-2.77	-26.236	514083.04	1863 23	168 41	25	2.9	1.6	38681.6943778
2875	OCT 15	10 39 28.82 1	7176.2404 6	0.071564 4	37	6662.68	51.6388 3	-1	121.1768 8	36	35.533 5	-3.02	-30.968	514115.43	1734 5	-45 10	41	3.4	1.1	38683.4440835
2900	OCT 17	4 38 53.85 2	7175.9666 6	0.071592 5	34	6662.22	51.6389 3	-2	113.9451 8	35	40.879 5	-3.30	-35.722	514144.86	1698 4	14 10	42	3.3	1.1	38685.1936789
2925	OCT 18	22 38 9.45 2	7175.6832 13	0.071605 5	30	6661.87	51.6398 4	-2	106.7141 8	34	46.205 7	-3.64	-40.498	514175.32	1869 8	111 20	30	3.2	1.4	38686.9431649
2950	OCT 20	16 37 14.64 2	7175.3562 12	0.071620 8	27	6661.46	51.6392 5	1	99.4801 9	34	51.530 12	-3.99	-45.317	514210.47	1834 20	-229 22	26	3.3	1.7	38688.6925306
2975	OCT 22	10 36 9.62 2	7175.0983 9	0.071630 7	25	6661.15	51.6399 5	4	92.2467 12	36	56.841 8	-4.26	-50.170	514238.19	1487 12	-51 20	36	3.1	1.9	38690.4417780
3000	OCT 24	4 34 56.26 2	7174.8886 8	0.071646 6	23	6660.84	51.6402 4	4	85.0146 12	39	62.171 7	-4.39	-55.094	514260.74	1210 13	13 15	33	3.5	1.6	38692.1909289
3025	OCT 25	22 33 35.63 3	7174.6747 11	0.071665 7	20	6660.50	51.6403 4	2	77.7815 14	39	67.516 9	-4.43	-60.087	514283.73	1540 9	124 17	26	3.3	1.7	38693.9399957
3050	OCT 27	16 32 6.11 2	7174.3993 6	0.071658 6	15	6660.30	51.6408 3	0	70.5503 11	37	72.840 7	-4.47	-65.122	514313.35	1650 8	-72 11	30	3.5	1.5	38695.6889596
3075	OCT 29	10 30 27.04 2	7174.1455 9	0.071637 6	10	6660.21	51.6417 4	2	63.3151 8	35	78.151 6	-4.61	-70.211	514340.64	1576 9	20 23	34	2.9	1.5	38697.4378130
3100	OCT 31	4 28 38.80 2	7173.8930 6	0.071613 4	4	6660.15	51.6425 4	4	56.0794 7	36	83.475 6	-4.75	-75.379	514367.79	1422 5	-89 10	36	3.2	1.4	38699.1865602
3125	NOV 1	22 26 42.06 2	7173.6686 15	0.071593 5	16	6660.08	51.6436 4	3	48.8428 8	38	88.789 8	-4.78	-80.603	514391.93	1598 28	168 34	22	2.8	1.2	38700.9352090
3150	NOV 3	16 24 36.13 3	7173.4042 27	0.071564 10	-3	6660.05	51.6417 6	3	41.6099 13	39	94.076 11	-4.91	-85.872	514420.37	1445 11	-80 29	16	3.7	1.6	38702.6837515
3175	NOV 5	10 22 21.47 3	7173.1836 8	0.071528 8	6	6660.10	51.6427 5	1	34.3747 14	38	99.414 9	-4.84	-91.264	514444.10	1349 13	6 17	25	4.0	1.8	38704.4321929
3200	NOV 7	4 19 58.60 3	7172.9622 8	0.071484 7	-9	6660.21	51.6430 4	0	27.1394 11	36	104.750 9	-4.67	-96.725	514467.92	1252 10	-82 15	29	3.1	1.8	38706.1805393
3225	NOV 8	22 17 28.01 2	7172.7615 6	0.071426 6	-14	6660.44	51.6436 4	1	19.9025 7	34	110.065 7	-4.48	-102.239	514489.51	1419 5	133 9	36	3.4	1.6	38707.9287964
3250	NOV 10	16 14 48.93 7	7172.5095 6	0.071370 4	-18	6660.61	51.6445 3	2	12.6636 7	33	115.390 7	-4.28	-107.833	514516.63	1430 5	-97 9	41	3.3	1.5	38709.6769552
3275	NOV 12	10 12 1.31 2	7172.2967 5	0.071312 4	-23	6660.83	51.6452 3	2	5.4254 7	34	120.729 6	-4.10	-113.512	514539.52	1308 6	-2 9	32	3.6	1.2	38711.4250152
3300	NOV 14	4 9 5.89 2	7172.0840 7	0.071252 7	-27	6661.06	51.6448 4	1	-1.8122 11	34	126.063 6	-3.95	-119.253	514562.41	1257 12	-39 14	17	2.9	1.0	38713.1729848
3325	NOV 15	22 6 2.80 5	7171.8763 14	0.071206 25	-30	6661.20	51.6437 6	-1	-9.0515 20	33	131.417 15	-3.80	-125.078	514584.55	1499 10	172 22	14	3.3	1.2	38714.9208657
3350	NOV 17	16 2 51.29 3	7171.6074 8	0.071055 7	-32	6662.03	51.6443 3	-1	-16.2859 9	30	136.719 7	-3.63	-130.915	514613.70	1592 5	-79 10	30	3.4	1.3	38716.6686492
3375	NOV 19	9 59 30.51 3	7171.3701 7	0.070976 7	-33	6662.37	51.6443 [illegible]	0	-23.5290 8	27	142.061 6	-3.32	-136.844	514639.25	1326 7	-82 13	36	3.1	1.4	38718.4163254
3400	NOV 21	3 56 2.15 3	7171.1776 11	0.070929 10	-34	6662.53	51.6448 6	2	-30.7692 11	26	147.410 9	-2.89	-142.829	514659.97	1143 13	8 18	25	3.4	1.7	38720.1639137
3425	NOV 22	21 52 27.08 3	7170.9762 19	0.070834 11	-37	6663.03	51.6450 7	3	-38.0091 12	27	152.762 8	-2.45	-148.865	514681.66	1445 13	117 29	28	3.3	1.8	38721.9114245
3450	NOV 24	15 48 43.91 3	7170.7084 9	0.070715 6	-39	6663.63	51.6448 4	0	-45.2472 11	28	158.123 8	-2.12	-154.949	514710.49	1453 8	-229 13	27	3.4	1.2	38723.6588416
3475	NOV 26	9 44 52.74 4	7170.5107 10	0.070617 5	-41	6664.15	51.6445 [illegible]	-1	-52.4871 10	24	163.489 11	-2.00	-161.067	514731.77	1346 16	112 21	35	3.0	1.7	38725.4061660
3500	NOV 28	3 40 54.27 4	7170.2728 8	0.070515 5	-42	6664.66	51.6451 [illegible]	0	-59.7306 8	23	168.864 9	-1.65	-167.218	514757.39	1440 10	-17 14	41	3.6	1.7	38727.1534059

Table 2 (Contd)

ORBITAL PARAMETERS OF ARIEL 2

Node	Date 1964	Time h m s	a	e	$10^4 e_1$	a(1−e)	i	100 i_1	Ω	100 Ω_1	ω	ω_1	M	n	n_1	$10^{-2} n_2$	N	D	ε	MJD
3525	NOV 29	21 36 48.28 4	7170.0515 5	0.070404 10	-41	6665.25	51.6457 6	2	-66.9722 10	22	174.230 3	-1.35	-173.374	514781.22	1318 9	0 14	33	3.2	1.5	38728.9005588
3550	DEC 1	15 32 35.39 2	7169.8315 8	0.070265 8	-40	6666.04	51.6457 5	3	-74.2130 6	22	179.598 5	-0.97	-179.538	514804.92	1357 10	-16 15	35	3.3	1.3	38730.6476319
3575	DEC 3	9 28 15.48 3	7169.6122 13	0.070184 8	-39	6666.42	51.6456 5	2	-81.4530 12	23	184.967 8	-0.56	-185.702	514828.54	1339 10	-10 20	34	3.4	1.2	38732.3946236
3600	DEC 5	3 23 48.74 4	7169.3990 12	0.070082 8	-39	6666.95	51.6455 6	0	-88.6947 12	24	190.384 8	-0.19	-191.910	514851.50	1289 15	14 23	34	3.3	1.5	38734.1415364
3625	DEC 6	21 19 15.66 3	7169.1914 12	0.069976 6	-38	6667.52	51.6459 6	-2	-95.9381 10	23	195.802 7	0.12	-198.100	514873.87	1278 16	37 24	25	3.0	1.2	38735.8883757
3650	DEC 8	15 14 36.46 2	7168.9672 10	0.069867 7	-38	6668.09	51.6456 5	-2	-103.1771 6	21	201.187 4	0.30	-204.228	514898.02	1335 8	-73 15	31	3.2	1.2	38737.6351442
3675	DEC 10	9 9 50.81 3	7168.7615 12	0.069760 7	-36	6668.67	51.6461 7	-1	-110.4230 6	19	206.597 5	0.63	-210.350	514920.17	1239 11	-31 20	40	3.3	1.6	38739.3818381
3700	DEC 12	3 4 59.54 2	7168.5779 10	0.069662 6	-34	6669.20	51.6454 5	1	-117.6669 9	18	2[illegible].026 6	0.88	-216.453	514939.96	1019 9	-39 15	36	3.2	1.0	38741.1284669
3725	DEC 13	21 0 3.89 4	7168.4129 18	0.069570 11	-31	6669.71	51.6447 4	1	-124.9045 12	18	217.452 8	1.14	-222.507	514957.74	1170 17	41 53	33	2.6	1.0	38742.8750450
3750	DEC 15	14 55 3.16 2	7168.2152 10	0.069487 6	-27	6670.12	51.6452 4	-1	-132.1493 9	19	222.877 6	1.40	-228.507	514979.04	1173 11	-34 16	37	3.2	1.0	38744.6215644
3775	DEC 17	8 49 57.34 3	7168.0236 14	0.069404 7	-25	6670.53	51.6449 7	-4	-139.3909 11	19	228.320 9	1.64	-234.469	514999.69	1231 11	19 21	37	3.4	1.4	38746.3680248
3800	DEC 19	2 44 46.27 2	7167.8288 13	0.069311 5	-23	6671.02	51.6450 7	-5	-146.6343 9	17	233.768 6	1.83	-240.374	515020.69	1145 12	41 23	40	3.2	1.3	38748.1144245
3825	DEC 20	20 39 30.26 2	7167.6401 10	0.069240 5	-20	6671.35	51.6434 6	-1	-153.8805 7	16	239.221 5	1.98	-246.219	515041.03	1102 9	-62 15	31	3.5	1.1	38749.8607669
3850	DEC 22	14 34 9.74 3	7167.4768 11	0.069209 10	-17	6671.42	51.6437 7	2	-161.1235 10	16	244.663 5	2.08	-251.988	515058.63	981 12	5 19	23	3.3	1.3	38751.6070572
3875	DEC 24	8 28 45.39 2	7167.3119 17	0.069147 8	-13	6671.71	51.6450 7	2	-168.3697 10	17	250.103 7	2.18	-257.682	515076.40	999 16	-42 26	27	3.4	1.3	38753.3533031
3900	DEC 26	2 23 17.28 3	7167.1611 16	0.069074 11	-9	6672.10	51.6444 8	-1	-175.6141 11	18	255.573 13	2.28	-263.332	515092.66	907 16	-49 22	22	3.5	1.1	38755.0995055
3925	DEC 27	20 17 45.73 2	7167.0175 10	0.069034 5	-6	6672.25	51.6453 6	2	177.1436 7	18	261.045 6	2.34	-268.916	515108.14	860 18	0 22	19	3.0	0.8	38756.8456681
3950	DEC 29	14 12 10.78 3	7166.8741 14	0.069015 8	-2	6672.25	51.6421 9	-2	169.8969 10	17	266.499 8	2.42	-274.412	515123.60	959 12	31 17	17	3.5	1.1	38758.5917914
3975	DEC 31	8 6 32.20 3	7166.7126 21	0.068998 8	2	6672.22	51.6435 8	0	162.6548 11	17	271.971 7	2.43	-279.853	515141.01	955 18	-13 38	22	3.1	1.4	38760.3378727
4000	JAN 2	2 0 49.67 2	7166.5549 13	0.068990 7	5	6672.13	51.6444 8	2	155.4076 10	19	277.427 6	2.38	-285.208	515158.02	1059 14	44 21	27	3.3	1.3	38762.0839082
4025	JAN 3	19 55 3.17 2	7166.3827 19	0.068963 7	9	6672.17	51.6463 8	3	148.1640 8	20	282.912 7	2.28	-290.521	515176.58	1059 13	20 30	28	3.2	1.4	38763.8298978
4050	JAN 5	13 49 12.27 2	7166.2133 10	0.068981 7	13	6671.88	51.6455 8	1	140.9181 8	22	288.375 6	2.15	-295.750	515194.85	1002 16	9 21	29	3.1	1.5	38765.5758364
4075	JAN 7	7 43 17.26 2	7166.0490 10	0.068981 7	16	6671.73	51.6427 6	-1	133.6728 8	21	293.816 6	2.04	-300.894	515212.57	1090 11	39 12	25	3.6	1.1	38767.3217276
4100	JAN 9	1 37 17.60 2	7165.8498 13	0.069013 5	22	6671.31	51.6447 5	-3	126.4290 7	21	299.255 5	1.92	-305.980	515234.05	1299 13	18 21	28	3.2	1.0	38769.0675648
4125	JAN 10	19 31 12.24 2	7165.6426 13	0.069041 6	24	6670.92	51.6442 7	-1	119.1795 8	20	304.714 6	1.81	-311.027	515256.40	1246 26	-88 32	27	2.8	1.2	38770.8133361
4150	JAN 12	13 25 1.73 2	7165.4519 15	0.069065 8	26	6670.57	51.6459 7	1	111.9343 9	22	310.195 7	1.64	-316.041	515276.97	1307 12	27 21	32	3.4	1.6	38772.5590478
4175	JAN 14	7 18 45.38 1	7165.2327 12	0.069086 8	28	6670.22	51.6450 6	2	104.6863 8	25	315.629 6	1.41	-320.964	515300.62	1348 17	-35 23	32	3.2	1.3	38774.3046919
4200	JAN 16	1 12 22.85 2	7165.0190 11	0.069112 6	30	6669.83	51.6444 5	0	97.4400 8	27	321.048 6	1.10	-325.832	515323.67	1317 9	2 14	33	3.5	1.2	38776.0502645
4225	JAN 17	19 5 54.19 1	7164.8020 10	0.069170 5	33	6669.21	51.6440 5	-3	90.1916 7	26	326.471 4	0.78	-330.666	515347.08	1438 9	71 16	38	3.2	1.2	38777.7957660
4250	JAN 19	12 59 18.81 2	7164.5667 9	0.069233 6	37	6668.54	51.6430 [illegible]	-4	82.9404 8	24	331.912 5	0.50	-335.482	515372.47	1399 10	8 15	39	2.9	1.3	38779.5411899
4275	JAN 21	6 52 36.64 1	7164.3278 11	0.069268 6	40	6668.07	51.6422 4	-2	75.6889 8	24	337.346 5	0.28	-340.262	515398.25	1698 15	117 32	29	2.9	1.0	38781.2865352
4300	JAN 23	0 45 46.13 4	7164.0212 22	0.069257 18	41	6667.86	51.6430 9	0	68.4379 18	26	342.729 16	-0.06	-344.973	515431.33	2117 31	134 41	13	2.9	1.1	38783.0317839
4325	JAN 24	18 38 45.74 2	7163.6956 15	0.069336 [illegible]	41	6666.99	51.6433 7	0	61.1924 11	29	348.167 6	-0.22	-349.717	515466.48	1815 12	-81 23	16	3.2	1.0	38784.7769183
4350	JAN 26	12 31 36.12 1	7163.4219 9	0.069378 5	41	6666.44	51.6425 4	-1	53.9387 7	31	353.563 5	-0.57	-354.410	515496.02	1734 22	42 23	30	3.0	1.0	38786.5219458
4375	JAN 28	6 24 17.89 2	7163.1259 9	0.069449 7	42	6665.65	51.6416 5	-3	46.6840 9	30	358.976 7	-0.98	-359.111	515527.97	1972 8	20 15	31	3.4	1.3	38788.2668738

Table 2 (Contd)

ORBITAL PARAMETERS OF ARIEL 2

Node	Date 1964	Time h m s	a	e	$10^4 e_1$	a(1−e)	i	$100\, i_1$	Ω	$100\, Ω_1$	ω	$ω_1$	M	n	n_1	$10^{-2} n_2$	N	D	ε	MJD
4400	JAN 30	0 16 49.80 2	7162.8066 9	0.069491 7	42	6665.06	51.6410 4	2	39.4337 10	30	4.373 7	-1.40	-3.796	515562.44	1965 12	-18 20	35	3.1	1.2	38790.0116875
4425	JAN 31	18 9 11.51 2	7162.4832 7	0.069507 6	43	6664.64	51.6409 3	-1	32.1815 8	30	9.771 5	-1.80	-8.486	515597.36	2087 6	52 11	41	3.5	1.0	38791.7563831
4450	FEB 2	12 1 22.40 1	7162.1369 6	0.069539 4	44	6664.09	51.6412 3	0	24.9270 5	31	15.150 5	-2.13	-13.168	515634.76	2153 5	10 9	37	3.3	1.0	38793.5009537
4475	FEB 4	5 53 22.01 1	7161.7771 7	0.069573 6	44	6663.51	51.6415 4	1	17.6694 7	34	20.523 7	-2.40	-17.859	515673.61	2421 7	110 11	37	3.4	1.1	38795.2453936
4500	FEB 5	23 45 9.07 1	7161.3712 7	0.069613 5	42	6662.85	51.6420 4	0	10.4147 9	36	25.890 7	-2.63	-22.565	515717.46	2475 10	3 14	40	3.5	1.5	38796.9896883
4525	FEB 7	17 36 42.67 2	7160.9286 9	0.069633 6	40	6662.29	51.6410 5	-2	3.1574 12	36	31.261 6	-2.88	-27.299	515765.27	3086 9	144 16	45	3.4	1.9	38798.7338272
4550	FEB 9	11 28 0.43 1	7160.4427 6	0.069652 5	36	6661.70	51.6411 3	-2	-4.0990 7	35	36.638 6	-3.20	-32.068	515817.77	2710 5	-122 10	37	3.4	1.3	38800.4777827
4575	FEB 11	5 19 3.60 1	7160.0549 7	0.069660 4	33	6661.27	51.6408 4	0	-11.3596 8	34	41.983 7	-3.61	-36.846	515861.84	2379 9	-86 12	40	3.2	1.5	38802.2215694
4600	FEB 12	23 9 54.02 2	7159.6797 7	0.069674 5	31	6660.84	51.6409 4	1	-18.6198 9	36	47.342 7	-3.99	-41.675	515900.23	2074 9	-31 13	40	3.5	1.7	38803.9652085
4625	FEB 14	17 0 33.07 2	7159.3462 8	0.069680 5	29	6660.48	51.6405 4	0	-25.8783 10	39	52.697 6	-4.27	-46.544	515936.28	2171 9	57 13	37	3.3	1.6	38805.7087161
4650	FEB 16	10 51 0.35 2	7158.9915 7	0.069691 6	27	6660.07	51.6405 3	-3	-33.1372 9	40	58.043 7	-4.40	-51.454	515974.64	2144 26	-96 26	32	3.0	1.3	38807.4520874
4675	FEB 18	4 41 15.93 1	7158.6732 7	0.069679 4	23	6659.86	51.6399 3	-4	-40.3999 7	38	63.389 6	-4.48	-56.418	516009.03	1882 6	-20 11	42	3.2	1.4	38809.1953232
4700	FEB 19	22 31 20.98 1	7158.3730 5	0.069682 4	18	6659.56	51.6397 3	-2	-47.6635 7	36	68.731 6	-4.67	-61.434	516041.49	1895 6	17 9	43	3.3	1.4	38810.9384373
4725	FEB 21	16 21 15.39 2	7158.0620 6	0.069662 5	13	6659.42	51.6395 4	0	-54.9267 9	36	74.072 6	-4.70	-66.510	516075.13	1991 8	35 11	43	3.4	1.6	38812.6814281
4750	FEB 23	10 10 58.78 2	7157.7450 6	0.069645 5	8	6659.24	51.6393 4	0	-62.1882 9	37	79.419 6	-4.84	-71.654	516109.41	2115 6	161 10	39	3.5	1.6	38814.4242914
4775	FEB 25	4 0 30.22 2	7157.3760 7	0.069605 5	4	6659.19	51.6393 3	0	-69.4534 7	38	84.755 6	-4.95	-76.857	516149.32	2260 5	-26 11	36	3.3	1.5	38816.1670165
4800	FEB 26	21 49 49.04 2	7157.0195 7	0.069570 6	0	6659.11	51.6390 4	-2	-76.7186 9	39	90.098 7	-4.99	-82.131	516187.89	2183 8	-14 12	32	3.6	1.7	38817.9095953
4825	FEB 28	15 38 55.57 3	7156.6666 11	0.069529 8	-3	6659.07	51.6377 5	-3	-83.9837 13	37	95.443 9	-4.97	-87.478	516226.07	2243 16	12 28	25	3.0	1.9	38819.6520320
4850	MAR 2	9 27 49.61 3	7156.3083 10	0.069472 7	-7	6659.15	51.6382 5	-1	-91.2484 11	35	100.799 7	-4.87	-92.908	516264.84	2378 27	140 32	27	3.0	1.6	38821.3943242
4875	MAR 4	3 16 30.27 3	7155.8721 14	0.069397 8	-12	6659.28	51.6387 7	2	-98.5178 14	33	106.137 9	-4.64	-98.392	516312.04	2963 12	55 20	35	3.4	2.5	38823.1364614
4900	MAR 5	21 4 54.87 2	7155.4010 7	0.069319 5	-16	6659.40	51.6392 4	3	-105.7869 8	33	111.484 6	-4.53	-103.954	516363.03	2711 7	-45 13	32	3.1	1.4	38824.8784128
4925	MAR 7	14 53 4.28 3	7154.9692 10	0.069229 7	-21	6659.64	51.6395 7	3	-113.0563 11	34	116.831 8	-4.35	-109.588	516409.78	2711 12	13 17	30	3.5	1.8	38826.6201884
4950	MAR 9	8 40 58.92 3	7154.5435 11	0.069136 7	-25	6659.91	51.6400 6	1	-120.3241 9	35	122.180 8	-4.19	-115.289	516455.87	2561 7	-19 15	34	3.4	1.9	38828.3617931
4975	MAR 11	2 28 39.48 3	7154.1486 11	0.069046 6	-28	6660.18	51.6402 6	-1	-127.5961 9	34	127.534 8	-4.05	-121.059	516498.63	2327 27	-53 37	31	3.0	1.9	38830.1032348
5000	MAR 12	20 16 6.96 4	7153.7605 12	0.068944 9	-30	6660.55	51.6392 5	0	-134.8669 11	31	132.886 10	-3.84	-126.890	516540.66	2554 17	03 20	22	3.2	1.6	38831.8445250
5025	MAR 14	14 3 20.48 3	7153.3305 8	0.068838 6	-33	6660.91	51.6399 5	4	-142.1363 12	30	138.254 8	-3.51	-132.794	516587.24	2734 20	77 21	28	2.8	1.5	38833.5856536
5050	MAR 16	7 50 19.07 3	7152.8791 7	0.068721 6	-36	6661.33	51.6409 4	6	-149.4105 10	30	143.629 7	-3.12	-138.759	516636.14	2658 11	-120 17	35	3.6	1.4	38835.3266096
5075	MAR 18	1 37 3.45 2	7152.4784 7	0.068604 5	-39	6661.79	51.6413 4	4	-156.6813 8	31	148.997 5	-2.75	-144.762	516679.56	2395 6	-48 10	42	3.4	1.3	38837.0674010
5100	MAR 19	19 23 34.95 2	7152.1067 8	0.068473 6	-41	6662.38	51.6419 4	0	-163.9557 10	30	154.375 5	-2.44	-150.818	516719.84	2279 7	6 13	38	3.3	1.2	38838.8080434
5125	MAR 21	13 9 54.25 3	7151.7313 8	0.068348 5	-42	6662.92	51.6424 4	-1	-171.2297 10	28	159.770 7	-2.18	-156.925	516760.52	2471 12	24 18	34	3.1	1.4	38840.5485446
5150	MAR 23	6 56 0.64 3	7151.3164 8	0.068222 5	-42	6663.44	51.6433 5	-1	-178.5058 8	27	165.162 6	-1.89	-163.056	516805.49	2757 9	124 14	36	3.1	1.5	38842.2888963
5175	MAR 25	0 41 52.80 3	7150.8553 8	0.068095 8	-42	6663.92	51.6426 5	1	174.2194 8	26	170.551 7	-1.54	-169.203	516855.48	2781 8	-83 14	36	3.3	1.4	38844.0290833
5200	MAR 26	18 27 30.66 3	7150.4230 9	0.067954 6	-42	6664.52	51.6424 5	3	166.9447 9	25	175.947 6	-1.13	-175.367	516902.35	2587 9	-103 15	41	3.4	1.5	38845.7691048
5225	MAR 28	12 12 55.38 3	7150.0354 5	0.067826 5	-43	6665.08	51.6437 5	2	159.6690 9	25	181.358 6	-0.70	-181.552	516944.39	2275 10	-88 6	43	3.1	1.5	38847.5089743
5250	MAR 30	5 58 8.69 3	7149.6950 12	0.067697 8	-43	6665.68	51.6433 6	-2	152.3919 12	22	186.780 7	-0.45	-187.744	516981.30	1968 26	-91 39	31	2.8	1.3	38849.2487117

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7	R. H. Gooding	A Pegasus computer programme for estimating errors in the ephemeris of a satellite and an application to Ariel 1. R.A.E. Technical Note Space 37 (A.R.C.25058), July 1963
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14	W. H. Guier R. R. Newton	The earth's gravity field deduced from the Doppler tracking of five satellites. Johns Hopkins University, Applied Physics Laboratory, TG-634, December 1964

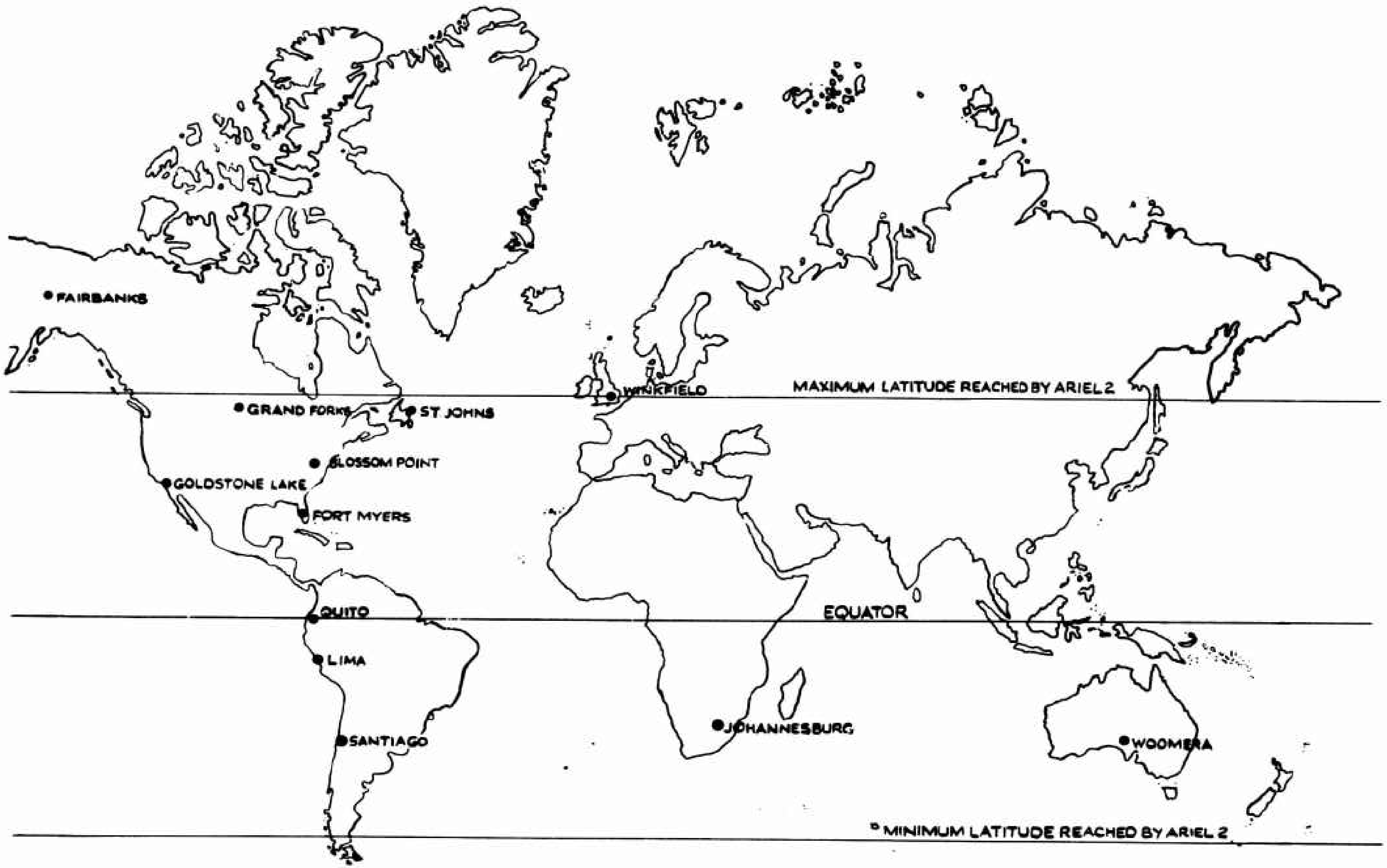


FIG.1 THE STADAN NETWORK OF MINITRACK STATIONS

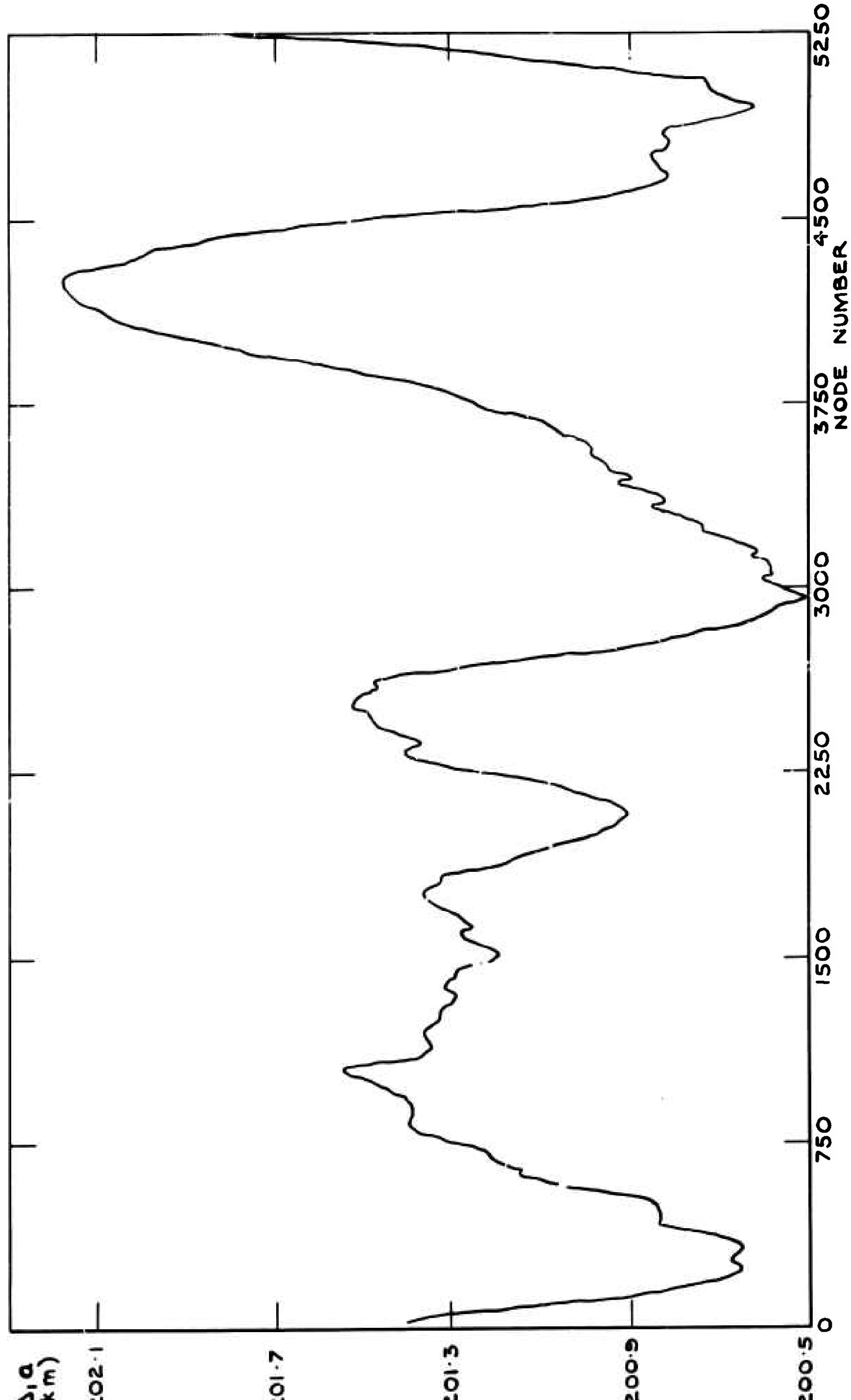


FIG. 2 SEMI-MAJOR AXIS, a, WITH QUINTIC POLYNOMIAL REMOVED

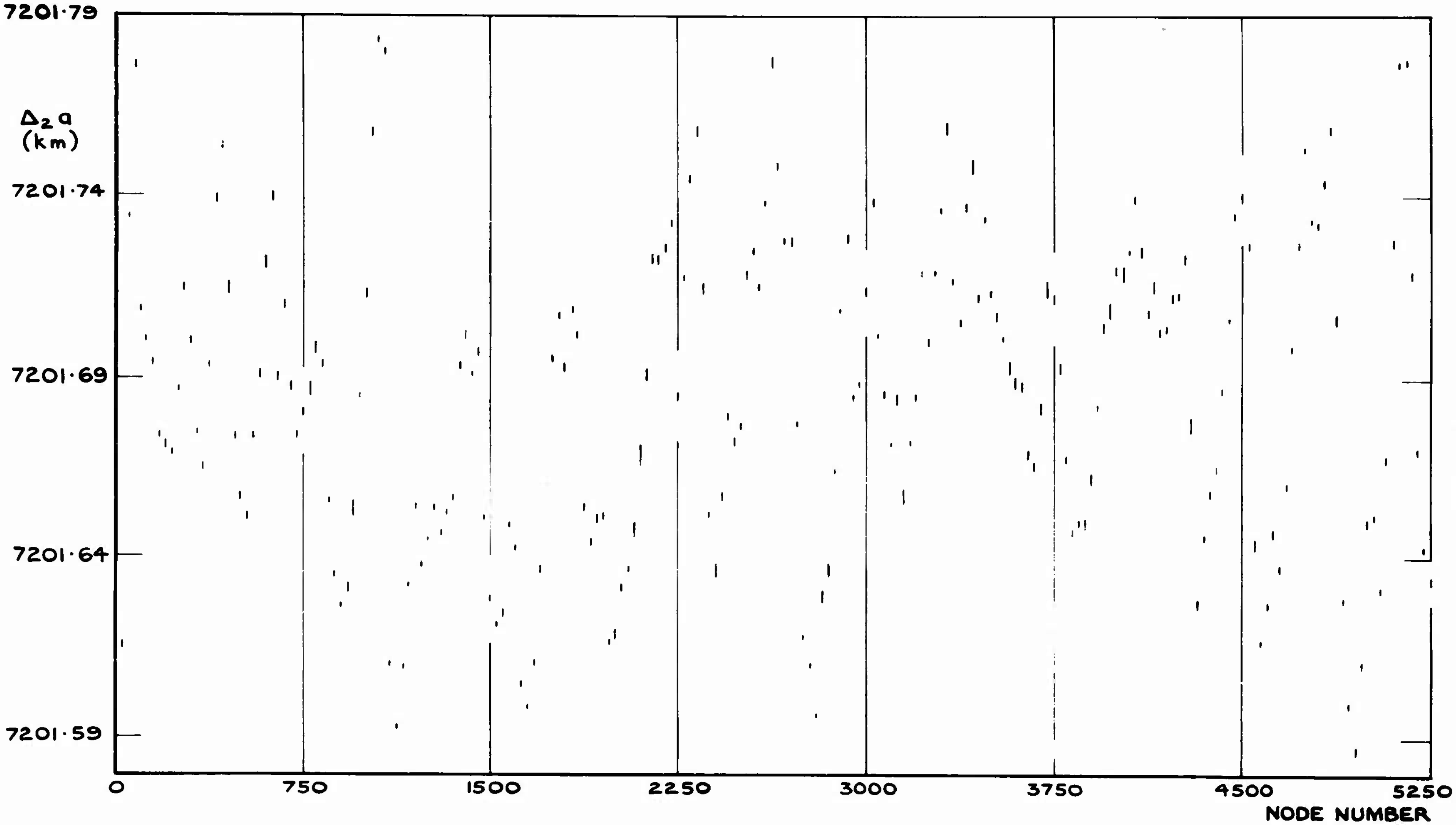


FIG. 3 SEMI-MAJOR AXIS, a, WITH QUARTIC POLYNOMIALS REMOVED IN SECTIONS

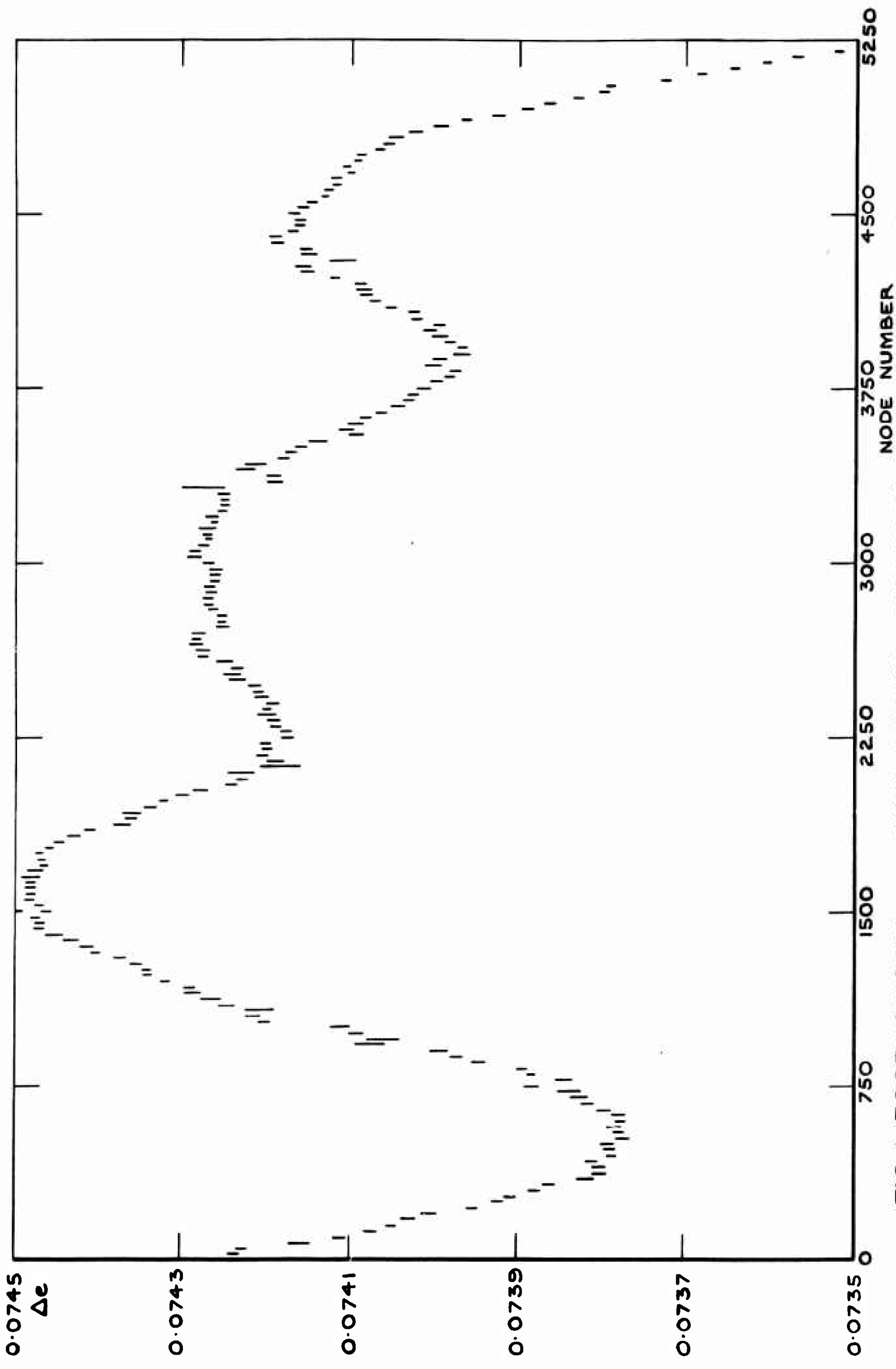


FIG.4 ECCENTRICITY, e , WITH LINEAR AND SINE TERMS REMOVED

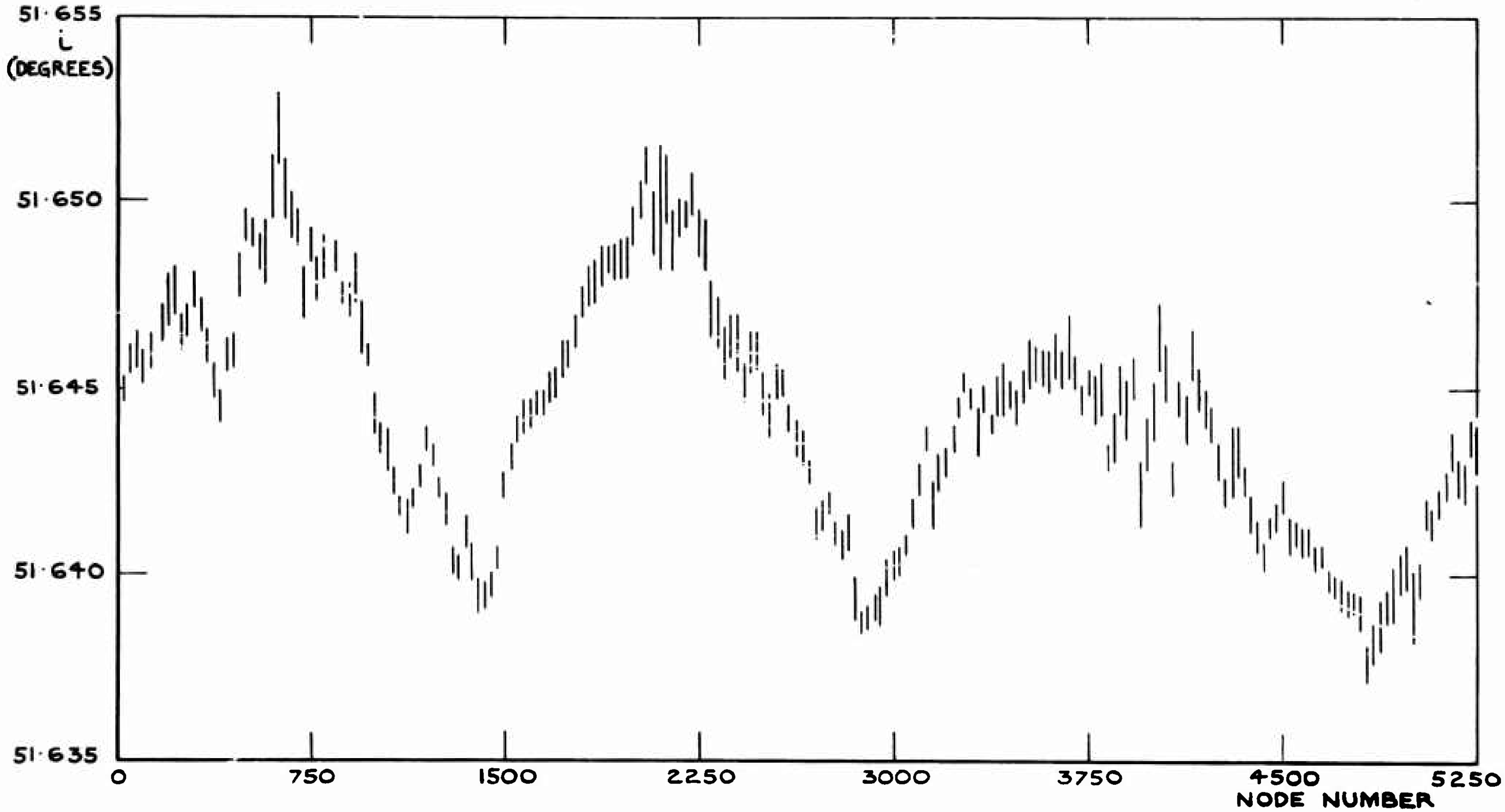


FIG. 5 ORBITAL INCLINATION i

Fig.6

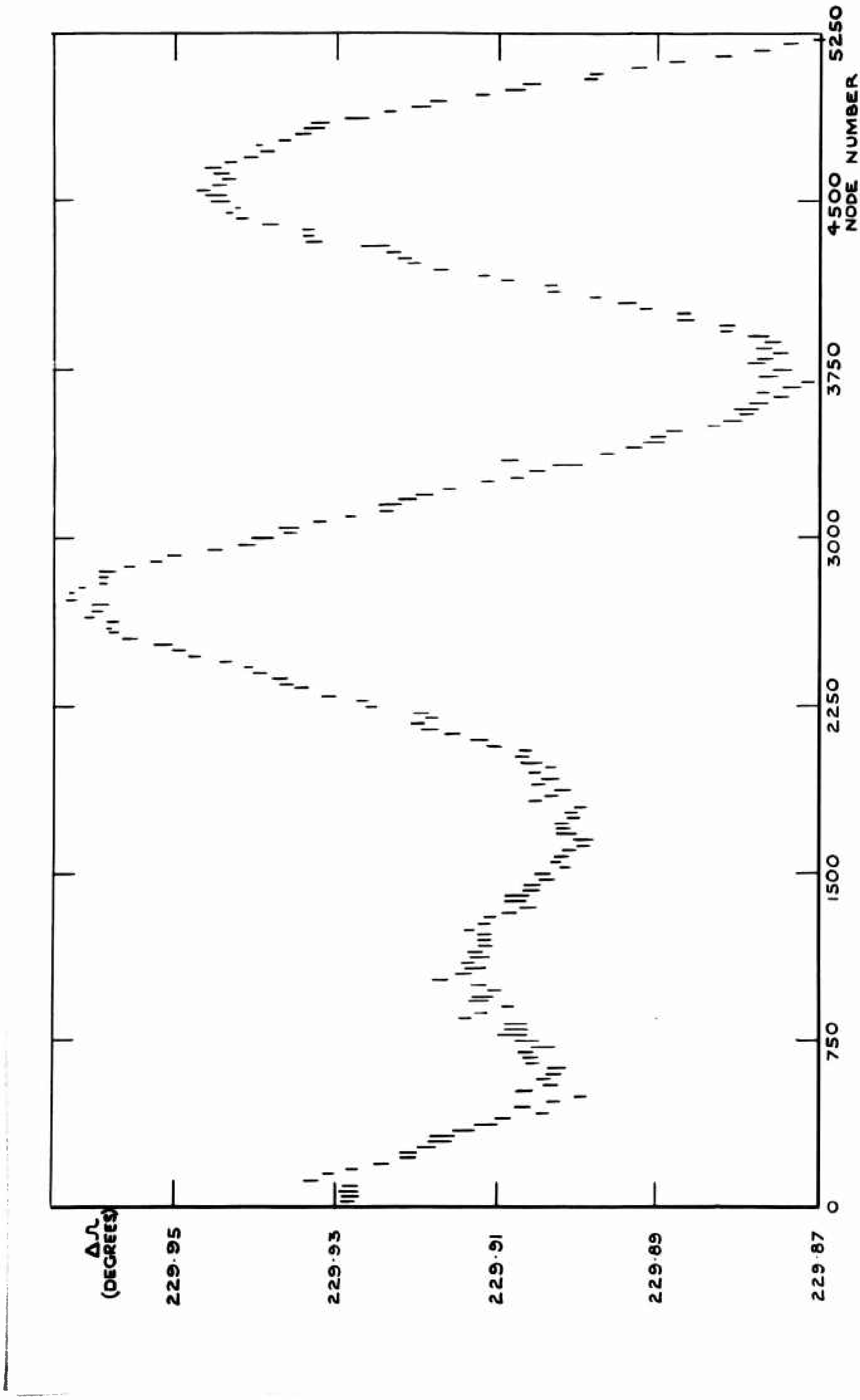


FIG. 6 RIGHT ASCENSION OF THE NODE, Ω, WITH CUBIC POLYNOMIAL REMOVED

SPA/P 1884

Fig.7

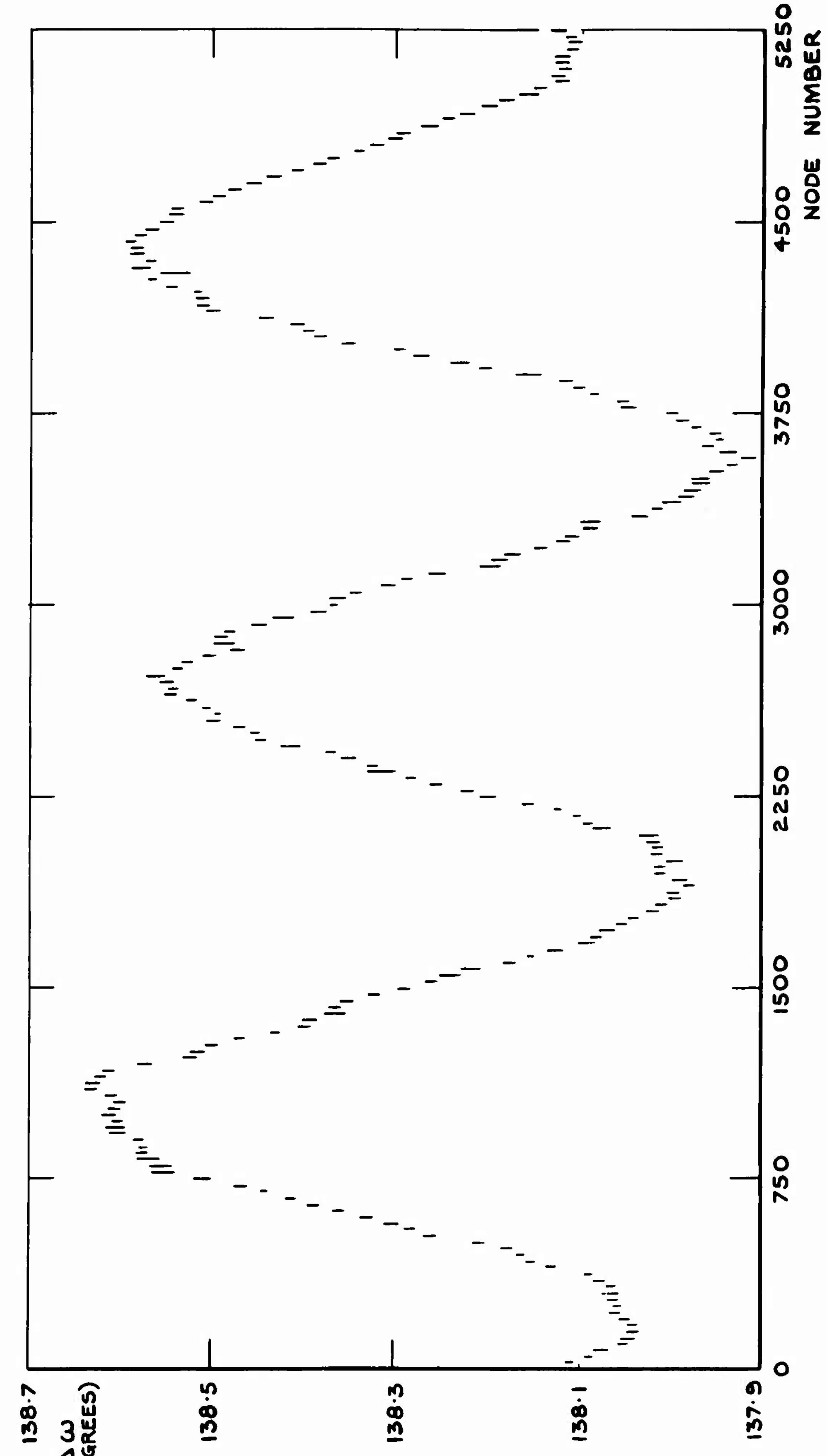


FIG.7 ARGUMENT OF PERIGEE, ω, WITH QUADRATIC AND COSINE TERMS REMOVED

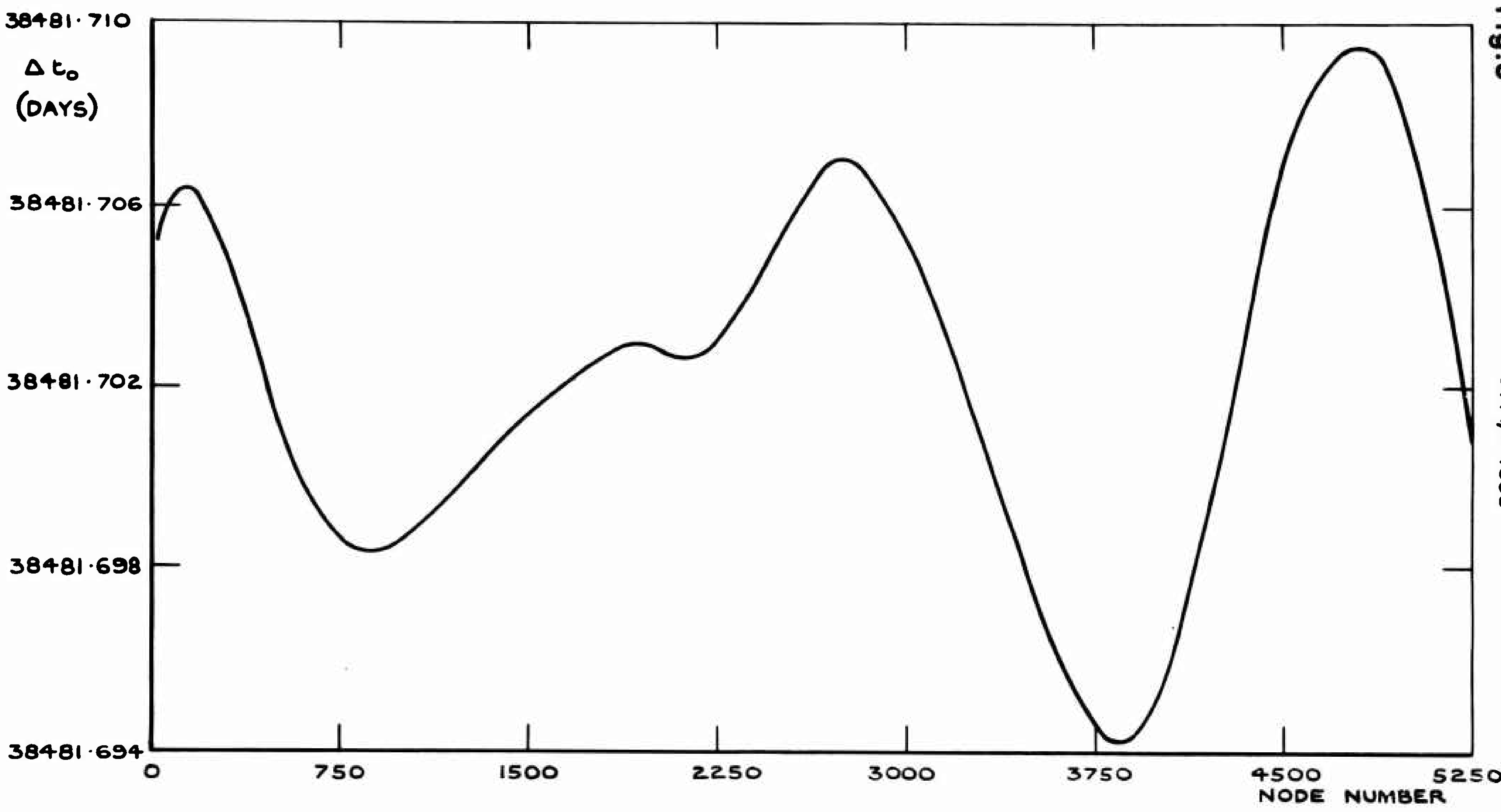


FIG. 8 EPOCH, t_o, WITH QUARTIC POLYNOMIAL REMOVED

Fig.9

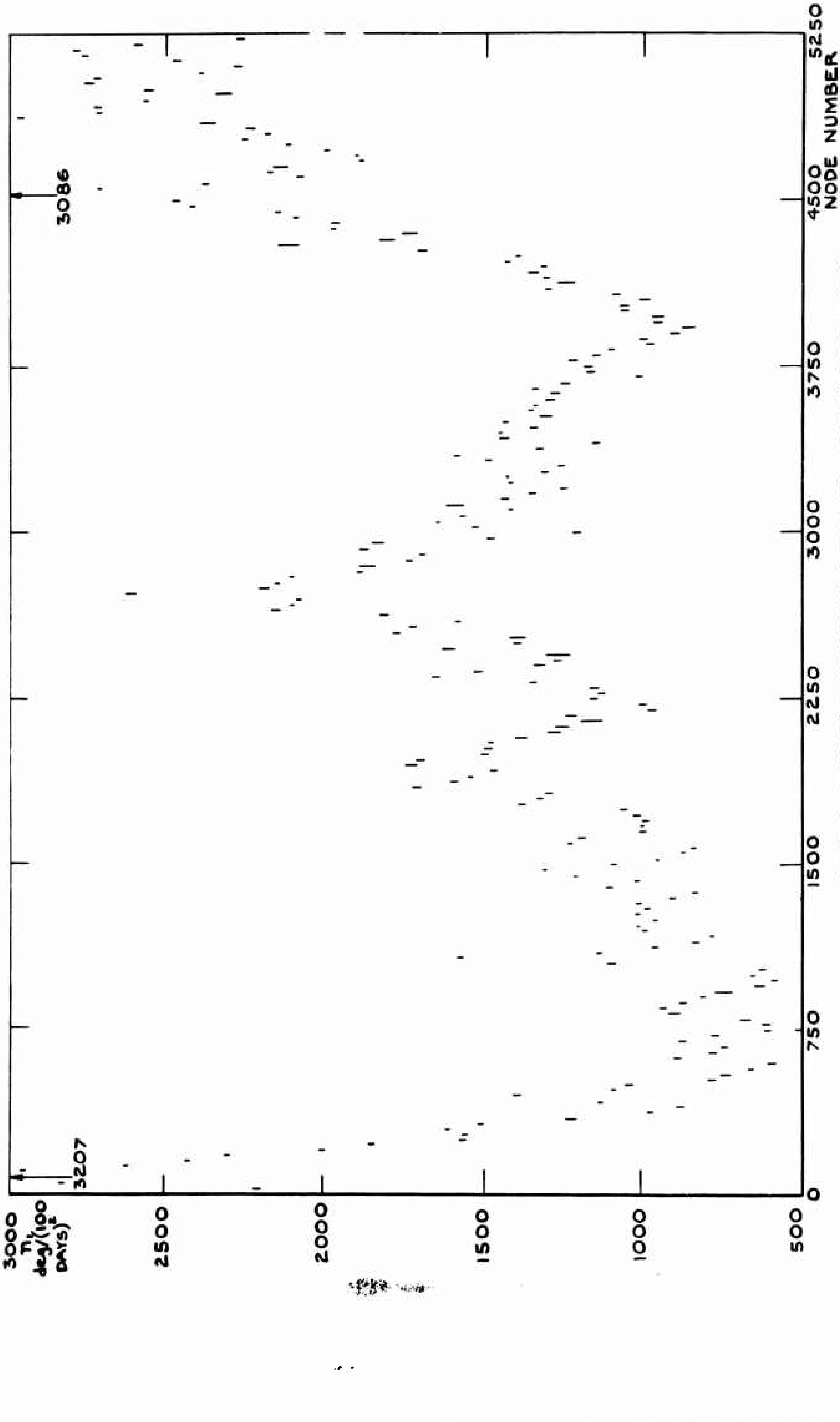


FIG.9 MEAN MOTION LINEAR COEFFICIENT n_1

Fig.10

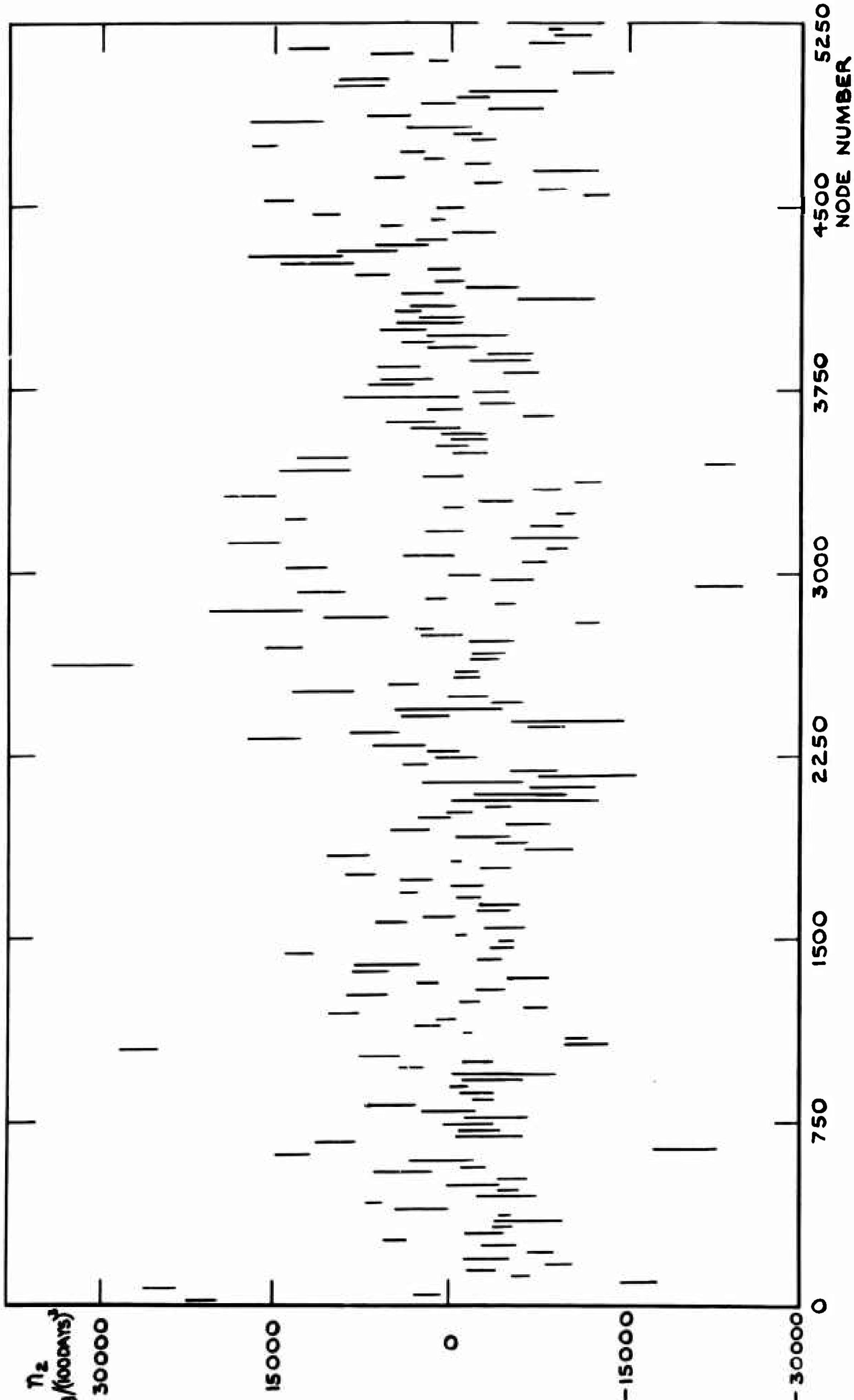


FIG.10 MEAN MOTION QUADRATIC COEFFICIENT n_2

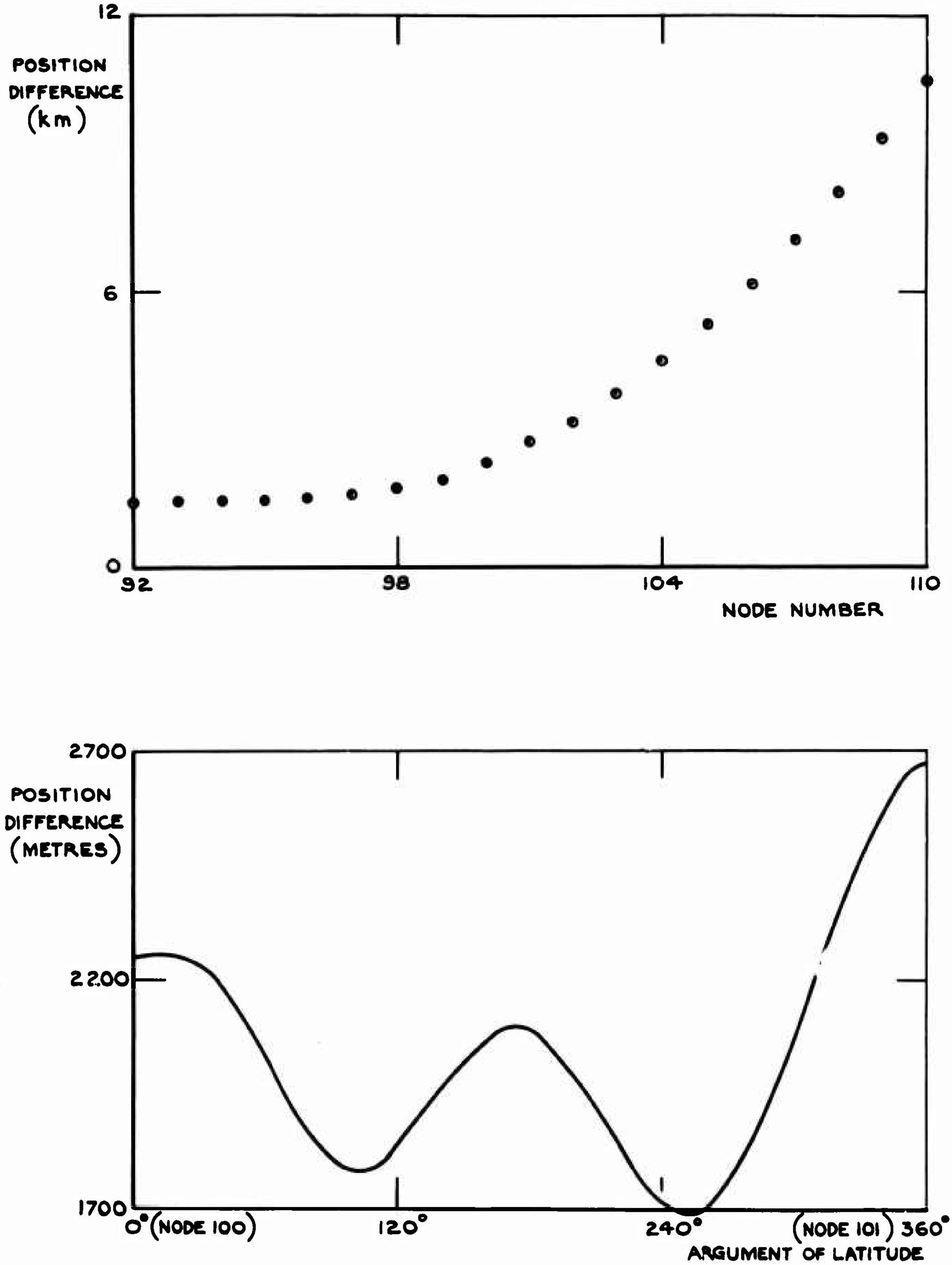
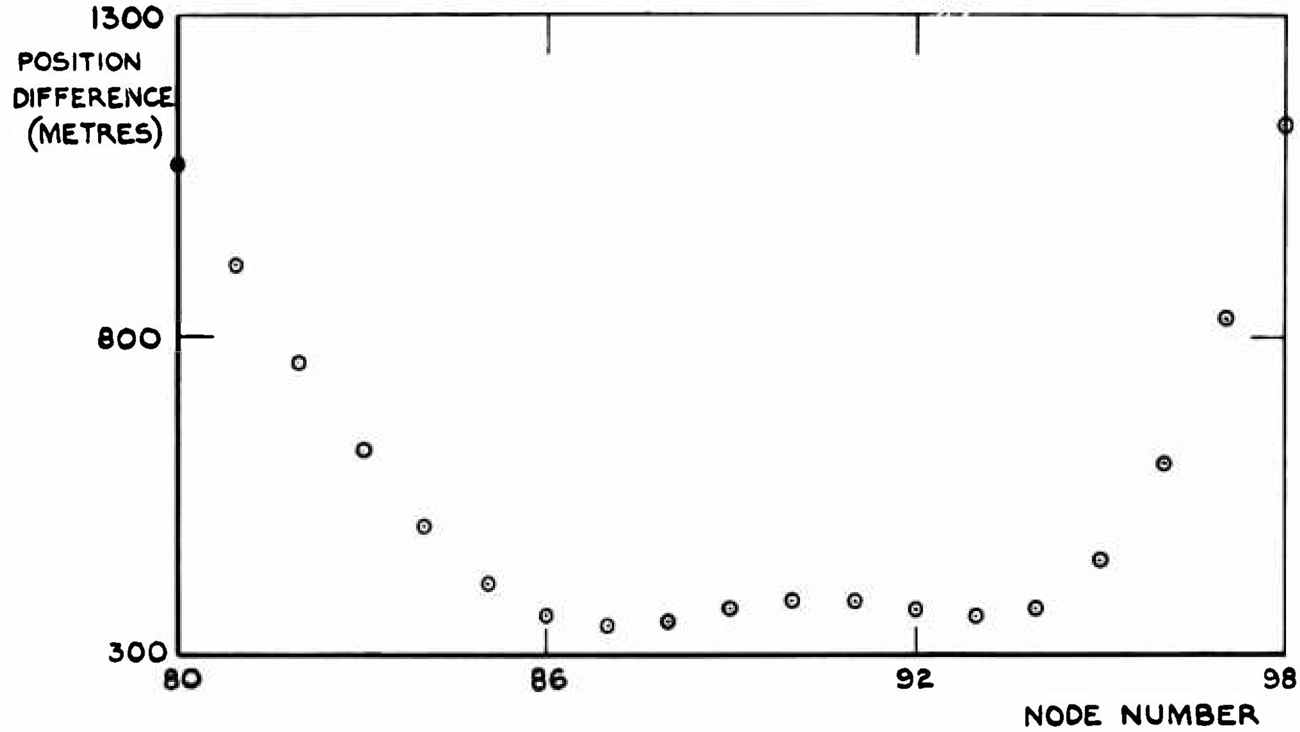


FIG. 11 DIFFERENCE BETWEEN COMPUTED SATELLITE POSITIONS BASED ON ORBITAL PARAMETERS FOR NODES 75 AND 125



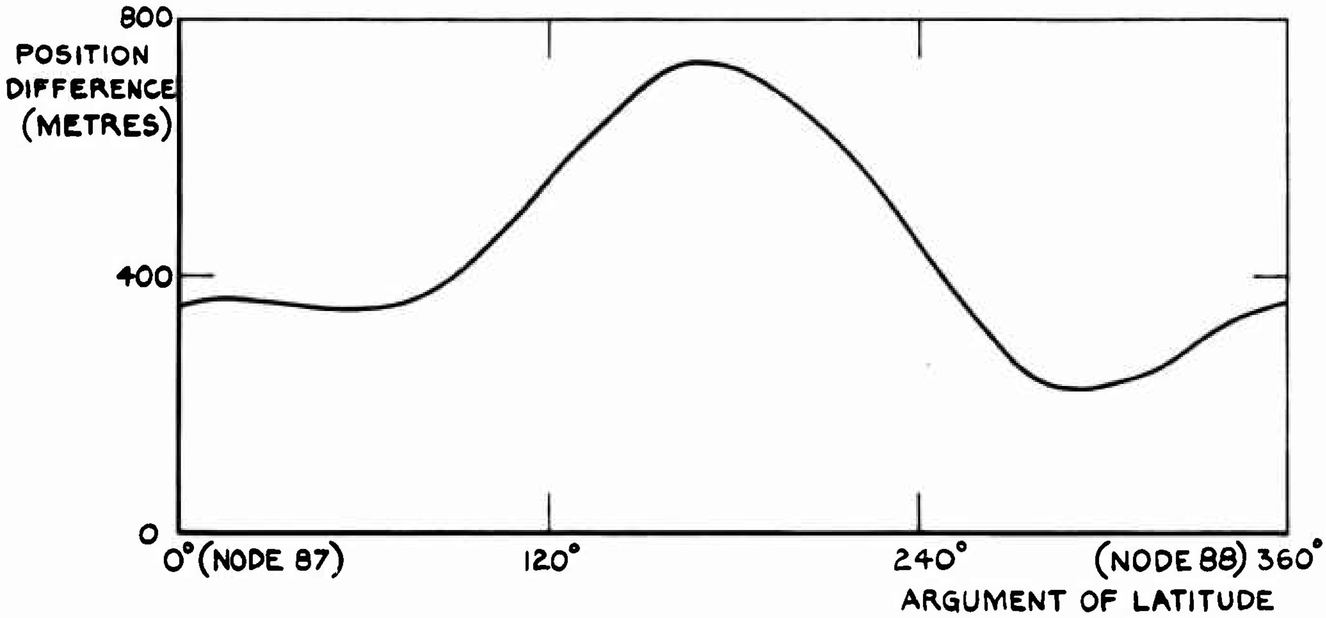
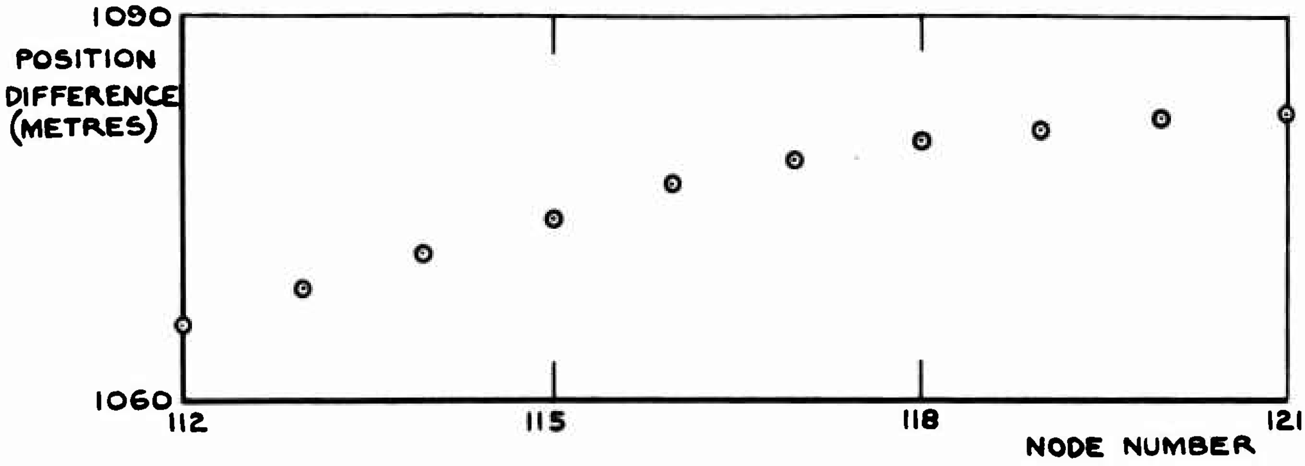


FIG.12 DIFFERENCE BETWEEN COMPUTED SATELLITE POSITIONS BASED ON ORBITAL PARAMETERS FOR NODES 75 AND 100



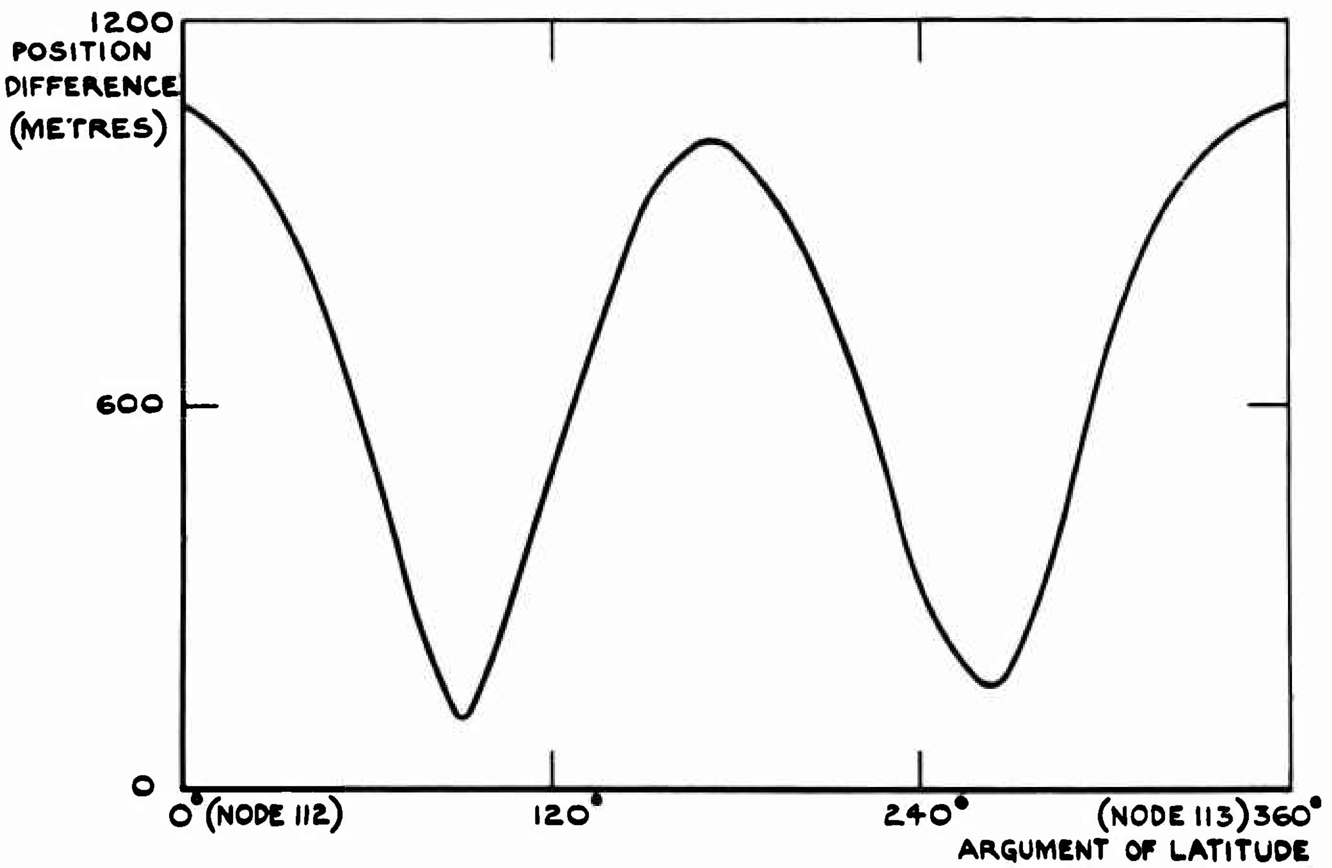


FIG.13 DIFFERENCE BETWEEN COMPUTED SATELLITE POSITIONS BASED ON ORBITAL PARAMETERS FOR NODES 100 AND 125

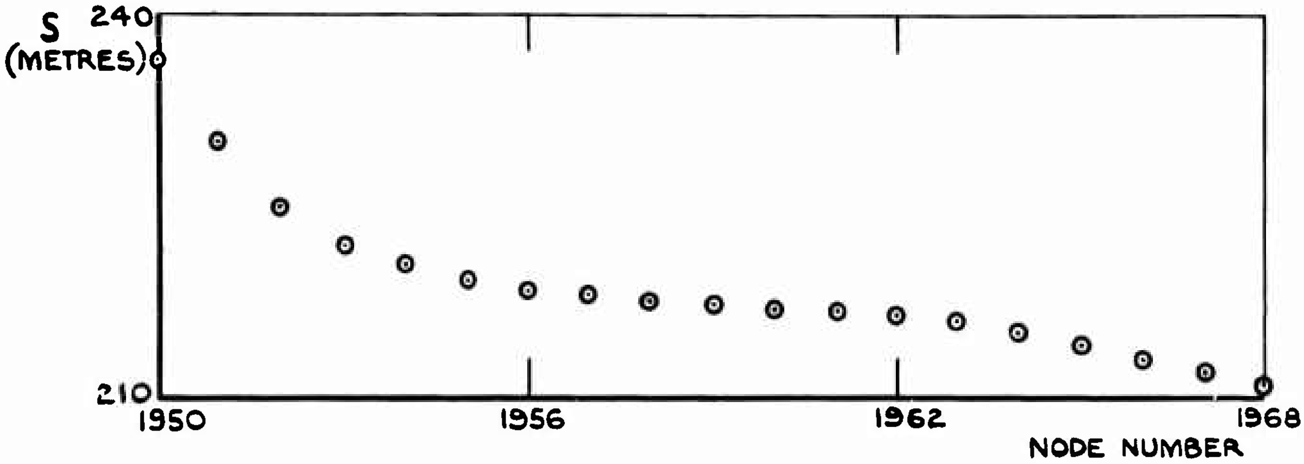


FIG. 14 PLOT OF S BASED ON COVARIANCE MATRIX FOR NODE 1950

d AND S ARE DEFINED ON FIG 16

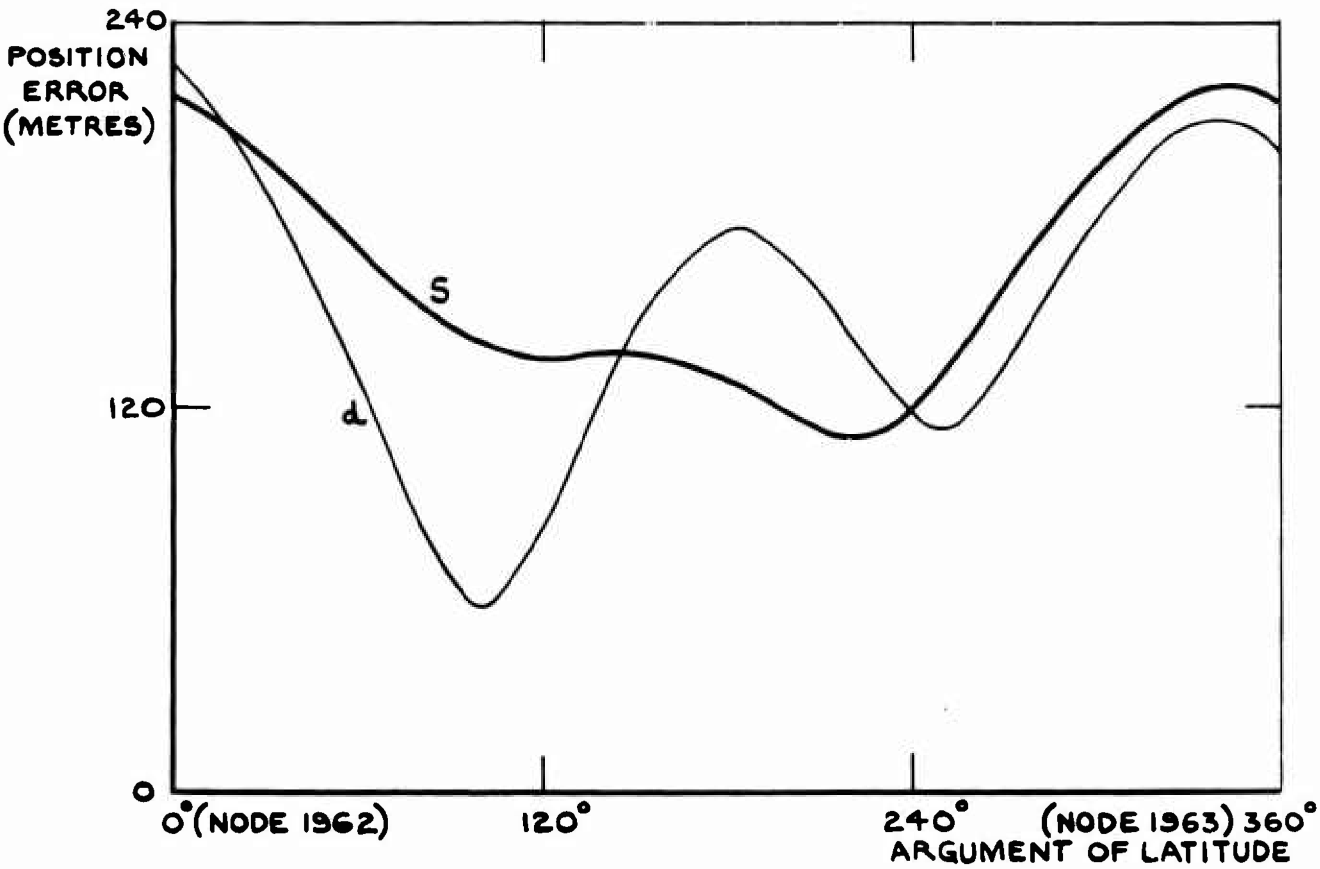


FIG. 15 ACCURACY OF COMPUTED SATELLITE POSITION
ASSOCIATED WITH PARAMETERS FOR NODES 1950 & 1975

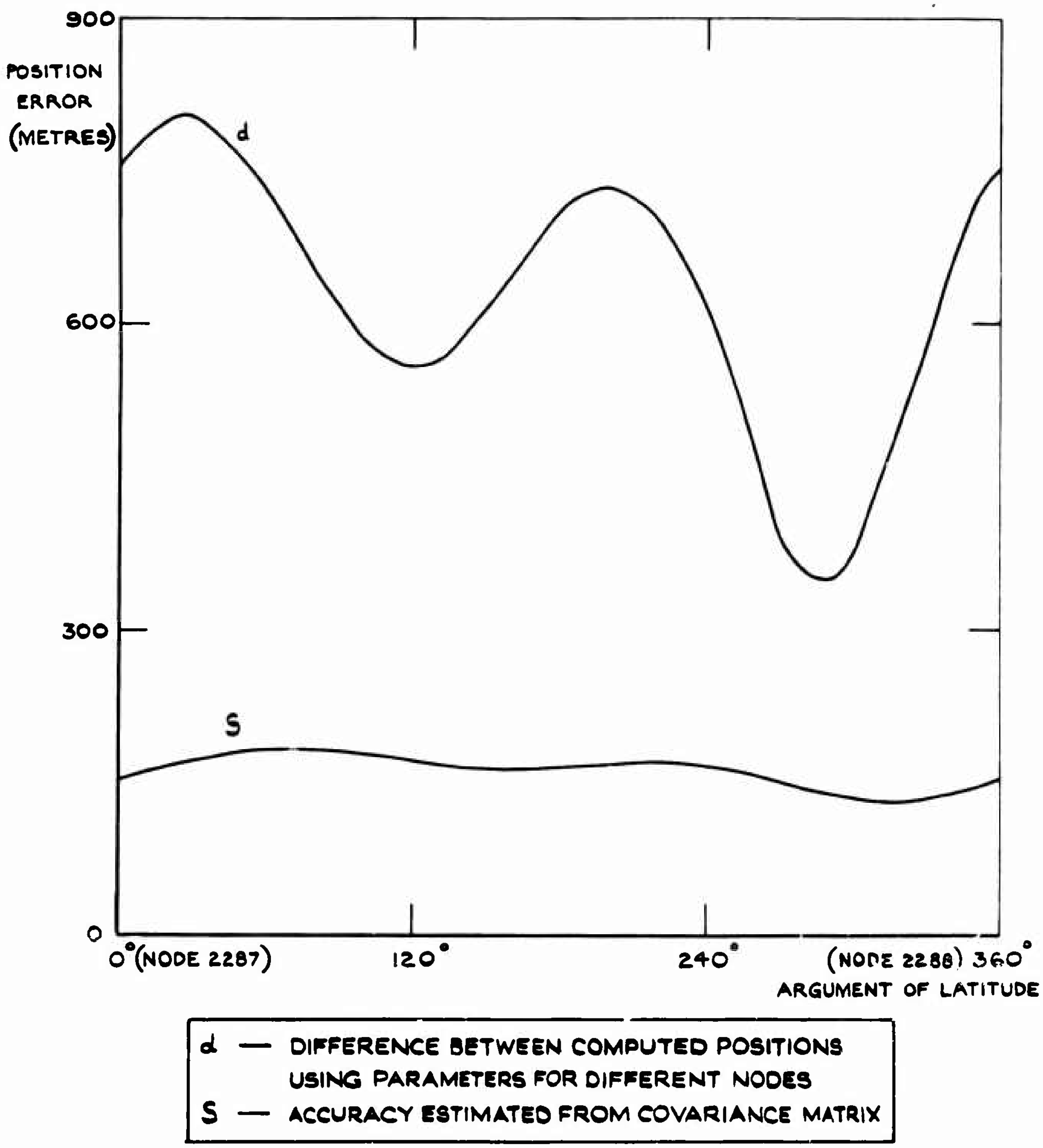
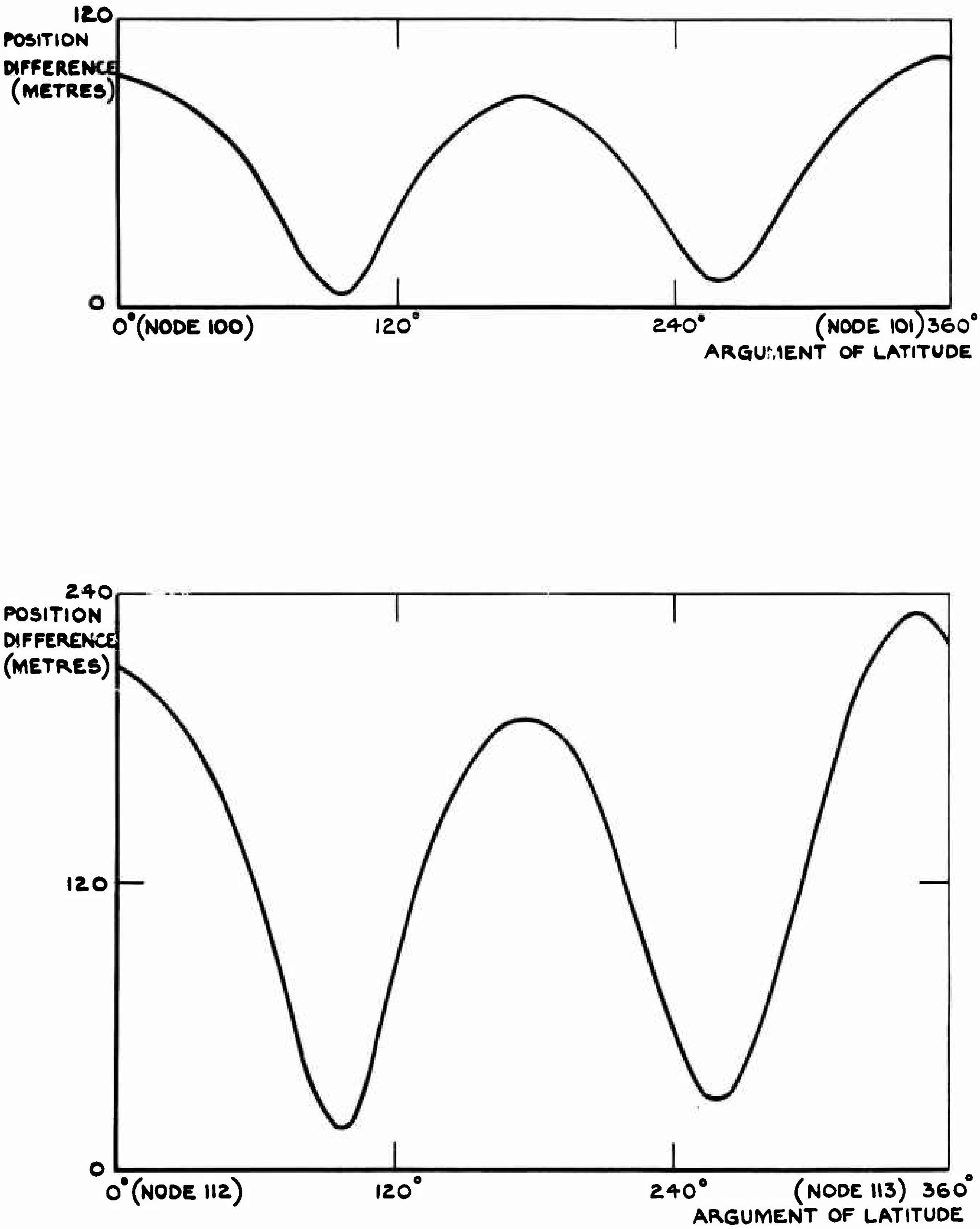
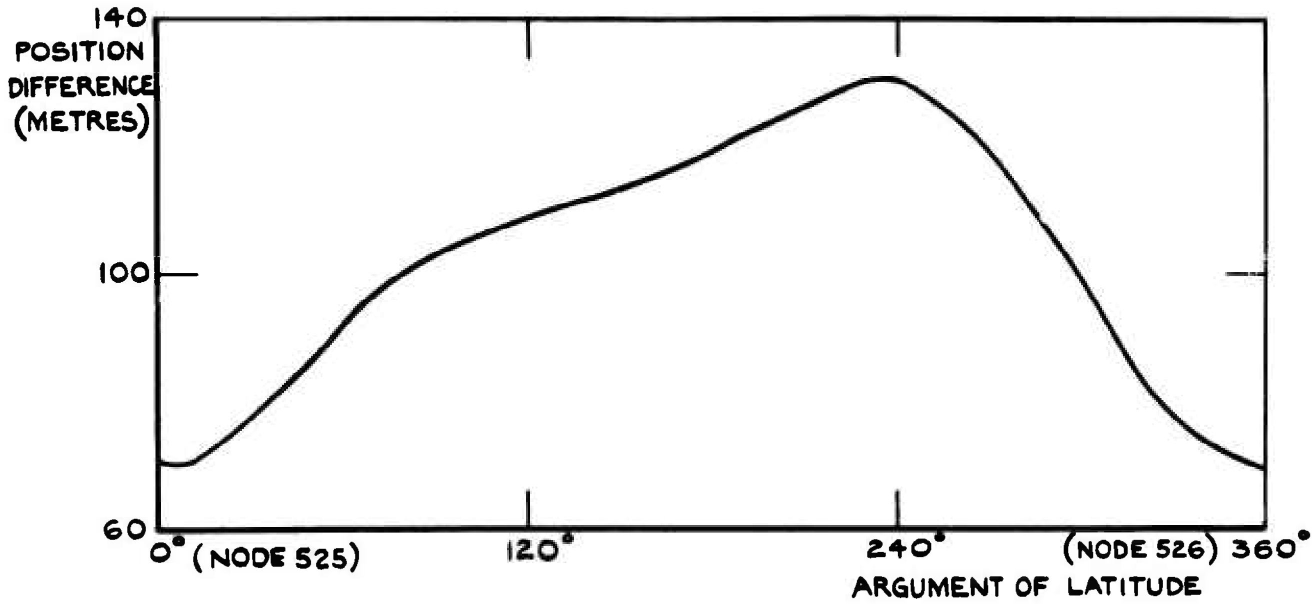


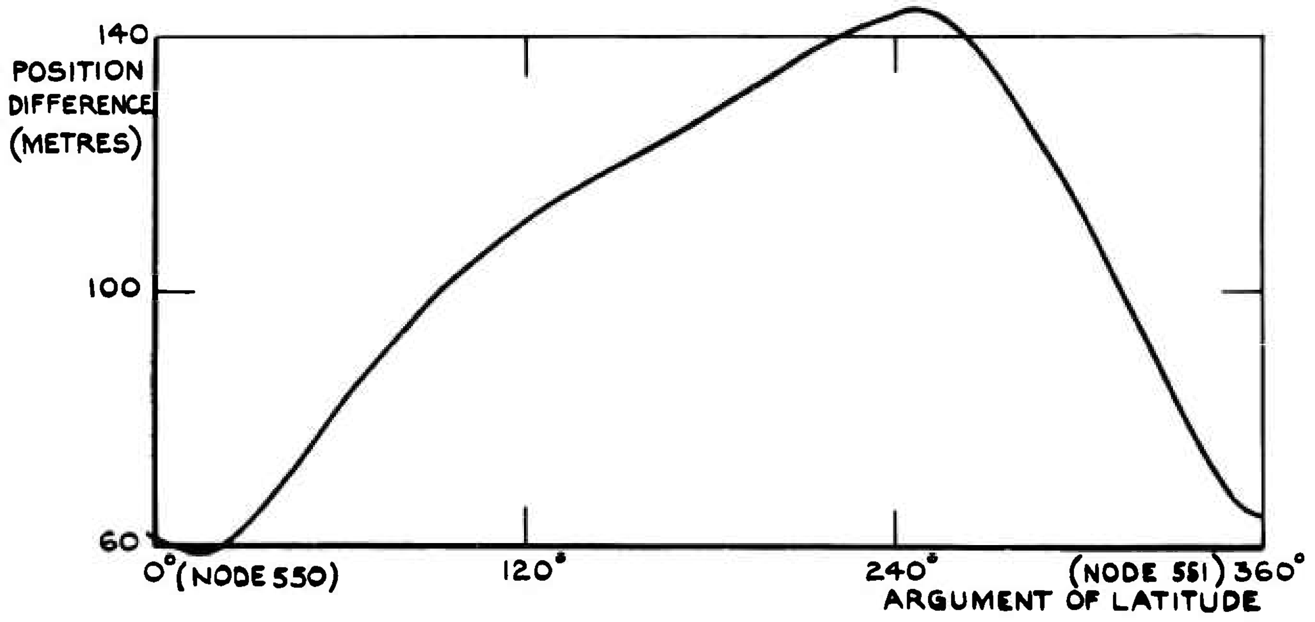
FIG 16 ACCURACY OF COMPUTED SATELLITE POSITION ASSOCIATED WITH PARAMETERS FOR NODES 2275 AND 2300



BOTH PLOTS BASED ON ORBITAL PARAMETERS FOR NODE 75

FIG. 17 VARIATION OF COMPUTED SATELLITE POSITION DUE TO DROPPING Ω_2 AND ω_2





BOTH PLOTS BASED ON ORBITAL PARAMETERS FOR NODE 525

FIG.18 VARIATION OF COMPUTED SATELLITE POSITION DUE TO ADOPTION OF FISCHER ELLIPSOID

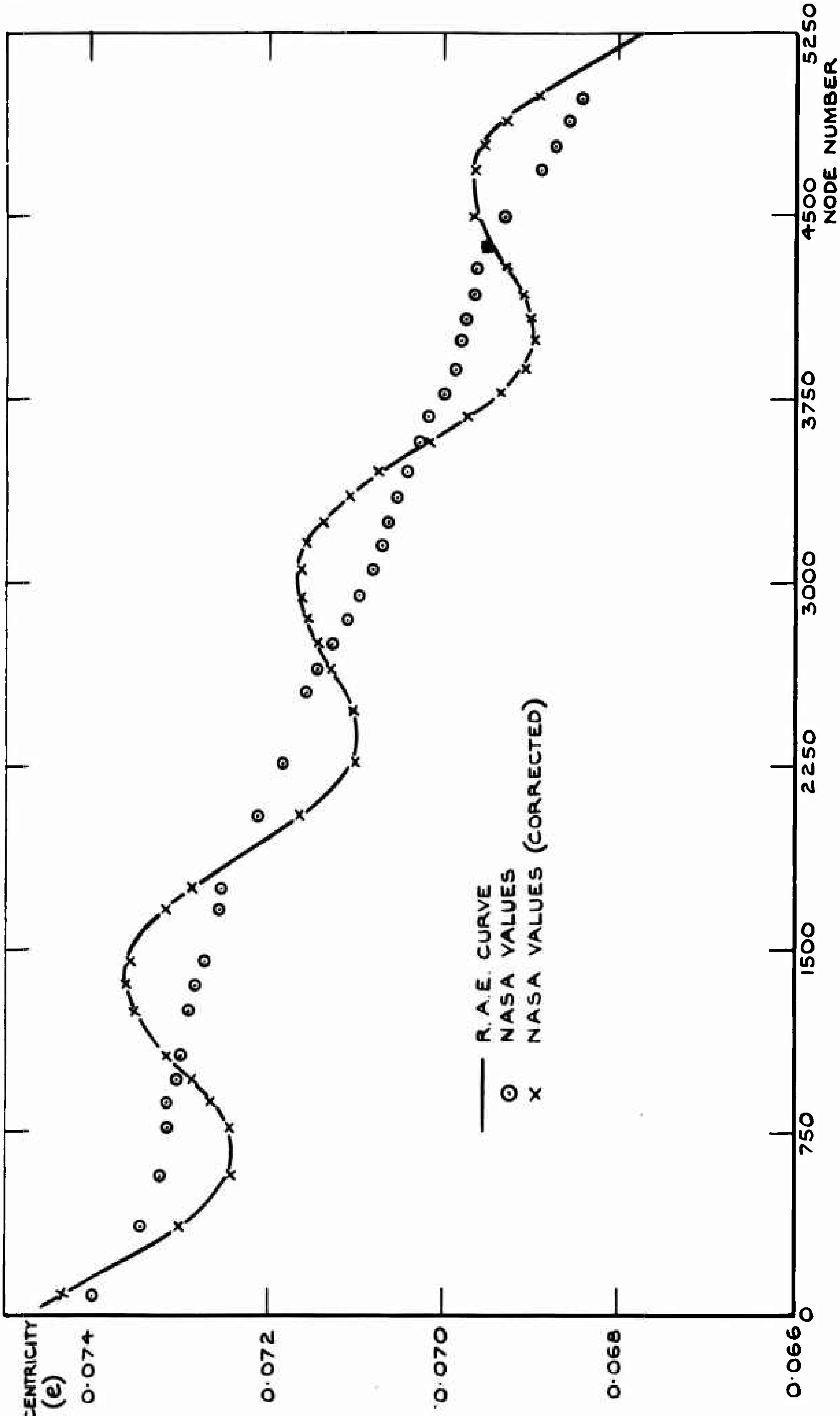
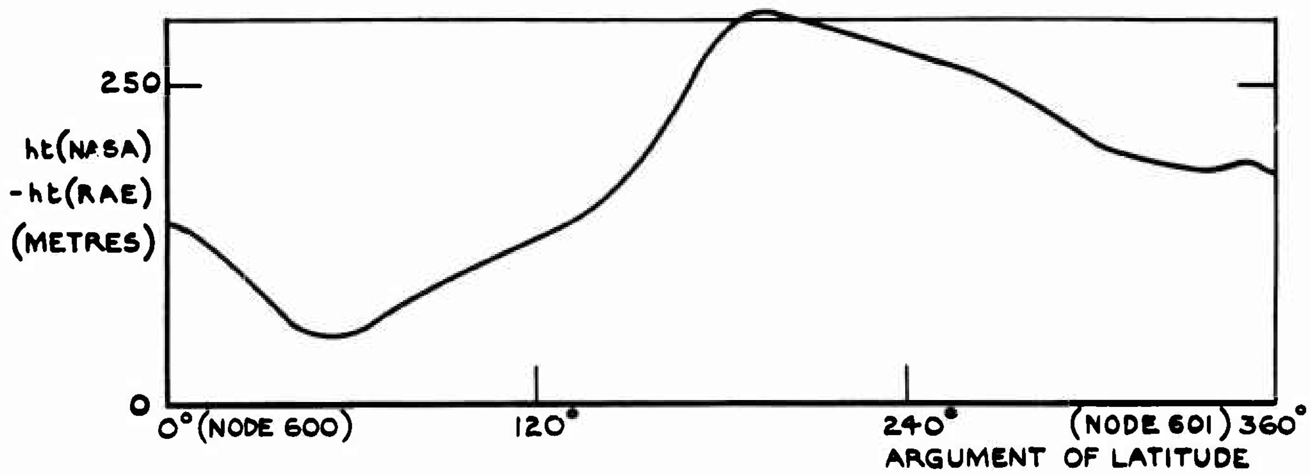


FIG.19 ECCENTRICITY OF NASA COMPARED WITH e (R.A.E.)



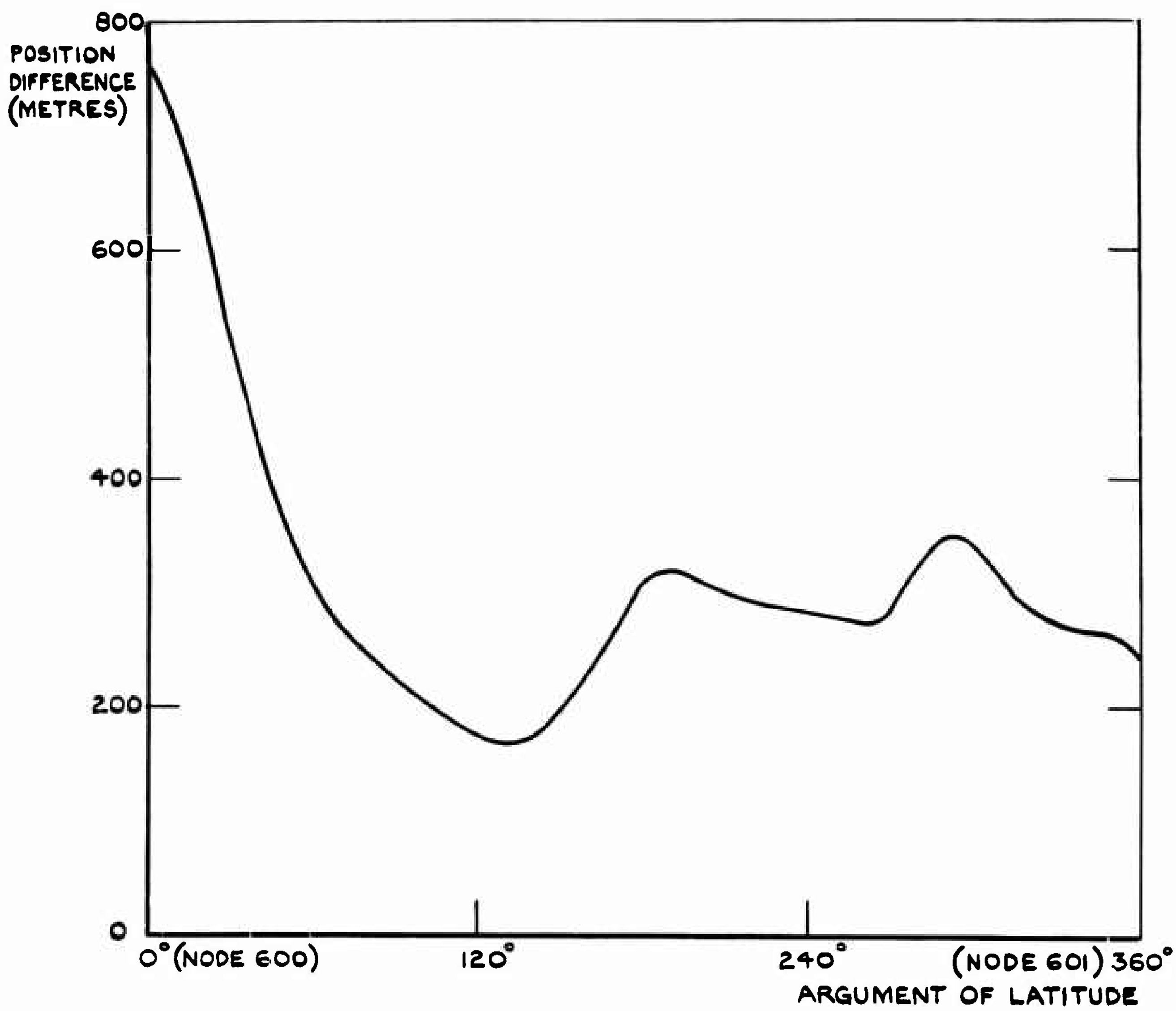


FIG. 20 COMPARISON OF NASA AND R.A.E. EPHEMERIDES

DETACHABLE ABSTRACT CARDS

Gooding, R. H. 521.6

THE ORBIT OF ARIEL 2 (1964-15A) - THE FIRST TWELVE MONTHS

Royal Aircraft Establishment Technical Report 65274 December 1965

The definitive orbit for Ariel 2 (1964-15A) is computed, from Minitrack observations, for a period of twelve months from the launch of the satellite. The orbit is described by a model with eight orbital parameters and these parameters are listed at every twenty-fifth nodal passage. The angular observations are accurate to about 1' and, as a result, the average computed standard deviations of the eight fitted orbital parameters are as follows: 1 m in semi-major axis, 10^{-5} in eccentricity, 2" in inclination, 4" in right ascension of the node, 30" in argument of perigee, $0^s.03$ in time at the node, and 0.001 deg/d^2 and 0.001 deg/d^3 in the linear and quadratic coefficients occurring in the mean motion polynomial.

(over)

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(over)

Ephemerides computed from the listed orbital parameters will be accurate to about $\frac{1}{2}$ km, the accuracy required by the Ariel 2 experimenters. Limitations which prevent the accuracy from being better than this are discussed.

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